1. (20%) Given an n×n matrix **A** which may not be symmetric. Answer the following questions and justify your answers:

(1) Is it true that the eigenvalues of A are all real?
(2) It it true that the matrix A exactly corresponds to n linearly independent eigenvectors?
(3) If A is invertible, is it true that det $A^{-1} = 1/\det A$?
(4) Under what condition the system $\mathbf{Ax} = \mathbf{0}$ has one–parameter solutions, where **x** is an n×1 unknown vector?
(5) Is it always possible to find a matrix **C** such that the product $\mathbf{C}^{-1}\mathbf{AC}$ is a diagonal matrix?

2. (20%) Explain the following terminologies:

(1) harmonic function,
(2) normal derivative,
(3) Stokes's theorem,
(4) Cauchy–Riemann equation,
(5) conservative force.

3. (20%) Find the Fourier series series of the periodic function of period 2π given by

$$f(x) = |x|, \quad |x| \le \pi,$$

and show that

$$\pi^2/8 = 1 + 1/3^2 + 1/5^2 + \cdots.$$

4. (20%) Evaluate the integral by complex variable methods

$$\int_{-\infty}^{\infty} \frac{\sin(x+a)\sin(x-a)}{x^2-a^2}\,dx$$

5. (10%) Let $\mathbf{F} = 2xe^{y}\mathbf{i} + (1+x^2e^{y})\mathbf{j}$. Calculate $\int_C \mathbf{F}\cdot d\mathbf{x}$, where C is a straight line from $(1,1) \to (-1,2)$ and then follows a parabolic route $y = x^2+1$ from $(-1,2) \to (2,5)$.

6. (10%) Write the general solutions of the partial differential equations:

$$\text{(i) } z_{xy} = 0; \quad \text{(ii) } z_{xx} - 3z_{xy} + 2z_{yy} = 0,$$

where the subscript indicates the partial derivative with respect to the corresponding space coordinate.