1. (20%) Explain the following terminologies:

(1) multiply connected region,
(2) normal derivative,
(3) divergence theorem,
(4) Laplace transform,

2. (20%) Solve the differential equation

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x^3.$$

3. (20%) Let $\mathbf{F} = (2xe^y)\mathbf{i} + (1+x^2 e^y)\mathbf{j}$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{x}$, where C is a straight line from $(1,1) \to (-1,2)$ and then follows a parabolic route $y = x^2+1$ from $(-1,2) \to (2,5)$.

4. (20%) Given an (n×n) matrix $\mathbf{A}$ which may not be symmetric

(1) It it true that the matrix $\mathbf{A}$ exactly corresponds to n linearly independent eigenvectors?
(2) Under what condition the system $\mathbf{Ax} = \mathbf{0}$ has one–parameter solutions, where $\mathbf{x}$ is an (n×1) unknown vector?
(3) Same as (ii), under what condition we have only trivial solution for $\mathbf{x}$?
(4) Is it always possible to find a matrix $\mathbf{C}$ such that the product $\mathbf{C^{-1}AC}$ is a diagonal matrix?

5. (20%) Expand the following function in a Fourier sine series of period 2ℓ, over the interval $(0,\ell)$

$$f(x) = \begin{cases} 0, & (x < 0) \\ x, & (0 < x < \ell/2), \\ \ell\text{-}x, & (x > \ell/2). \end{cases}$$