1. Given the Bessel's equation as

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-\frac{1}{4})y=0, \qquad (20)$$

and one of the solutions $\cos x/\sqrt{x}$, please find the other one to complete the final solution.

2. Given $w=3x^2y-y^3+y^2$, evalute the following integral

$$I=\oint_C \frac{\partial w}{\partial n}ds, \qquad (20)$$

where C is the elliptic curve $25x^2+y^2=25$, and n denotes the outer normal direction of C.

3. Evaluate the following integrals.

$$I=\int_{-\infty}^{\infty}\frac{dx}{(1+x^2)^n}, \qquad (20)$$

4. Solve the following partial differential equation.

$$\frac{\partial^2\phi}{\partial t^2}=\frac{\partial^2\phi}{\partial x^2} \qquad t>0 \quad x>0, \qquad (20)$$

with the following conditions

$$\phi(x,0)=0, \qquad \frac{\partial\phi(x,0)}{\partial t}=1, \quad x\geq 0$$

and

$$\phi(0,t)=\sin t, \quad \lim_{x\to\infty}\phi(x,t) \quad \text{is} \quad \text{finite.}$$

5. The strain energy per unit volume for a two dimensional elastic material can be represented as

$$u=\frac{1}{2E}(\sigma_{xx}^2+\sigma_{yy}^2)-\frac{\nu}{E}(\sigma_{xx}\sigma_{yy})+\frac{1}{2G}(\tau_{xy}^2), \qquad (20)$$

where $\nu\leq 0.5$ is the Poisson ratio of this material. If we rewrite the strain energy in matrix form as

$$u=\mathbf{xAx^T}$$

where

$$\mathbf{x}=(\sigma_{xx} \quad \sigma_{yy} \quad \tau_{xy})$$

and

$$\mathbf{A}=\begin{pmatrix}\frac{1}{2E} & \frac{-\nu}{2E} & 0\\ \frac{-\nu}{2E} & \frac{1}{2E} & 0\\ 0 & 0 & \frac{1}{2G}\end{pmatrix}$$

Please prove $u>0$ if $\mathbf{x}\neq\mathbf{0}$.