1. Explain the following terminologies (15)
   a. Wronskian
   b. Unitary matrix
   c. positive definite matrix

2. The solution of Legendre's differential equation

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0, \qquad (15)$$

is

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n[(x^2-1)^n]}{dx^n}$$

where n is an integer, prove

$$\int_{-1}^{1} P_m(x)P_n(x)dx = 0$$

if $m \neq n$

3. Find the solution of the following inverse Laplace transform

$$L^{-1}\{\ln\frac{s^2+1}{(s-1)^2}\}, \qquad (15)$$

4. Given a velocity field as

$$\vec{v} = y\vec{i} - z\vec{j} + yz\vec{k}$$

find the surface integral

$$I = \int\int_S \vec{v} \bullet \vec{n} dA, \qquad (15)$$

where $\vec{n}$ is a unit normal vector in the outer direction of the surface

$$S : x = \sqrt{y^2+z^2}; \qquad y^2+z^2 \leq 1$$

5. Find the particular solution of

$$y'' + 8y = f(t), \qquad (20)$$

where

$$f(t) = \begin{cases} \frac{\pi}{2} - t, & 0 \leq t \leq \pi \\ \frac{\pi}{2} + t, & -\pi < t \leq 0 \end{cases}$$

and $f(t+2\pi) = f(t)$

6. Solve the following wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}, \qquad (x>0, t>0), \qquad (20)$$

with the following conditions

$$y(x,0) = xe^{-x}$$

$$\frac{\partial y(x,0)}{\partial t} = 0$$