

1.(20%) Explain the terminologies

- (i) Hermitian matrix
- (ii) Stokes's theorem
- (iii) harmonic functions
- (iv) Fourier series.

2.(20%) A circular helix in space is given by

$$x = a \cos t, \quad y = a \sin t, \quad z = ct,$$

where  $a$ ,  $b$  and  $c$  are some constants and  $t$  is an arbitrary parameter

- (i) find the arc length from  $t = 0$ , to  $t = \pi/2$ .
- (ii) find the radius of curvature at  $t = \pi/3$ .

3.(20%) Derive the complete solutions for the differential equation

$$dy/dx + a(x)y = f(x).$$

4.(20%) Minimize  $x^2 + 4y^2 + 16z^2$  under the constraint  $xyz = 1$ .

5.(20%) Given a matrix  $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 7 & 0 \\ -2 & 4 & -5 \end{bmatrix}$ ,

- (i) Find the eigenvalues of the matrix  $A$
- (ii) Find all corresponding independent eigenvectors.