(1) The temperature $u(r)$ in the circular ring shown in Fig.1 is determined from the boundary-value problem

ODE: $r\dfrac{d^2u}{dr^2}+\dfrac{du}{dr}=0$,

BC: $u(a)=u_a$, $u(b)=u_b$.

where u_a and u_b are constant.
Solve for $u(r)$.

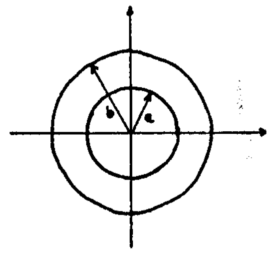

Fig. 1

(20%)

(2) Show that the equation

$$\sin\vartheta\frac{d^2y}{d\vartheta^2}+\cos\vartheta\frac{dy}{d\vartheta}+n(n+1)(\sin\vartheta)y=0$$

can be transformed in Legendre's equation by means of the substitution $x=\cos\vartheta$.

(20%)

(3) Solve the system

$$\begin{Bmatrix} dx_1/dt \\ dx_2/dt \end{Bmatrix}=\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}+\begin{Bmatrix} 3e^t \\ e^t \end{Bmatrix},$$

by diagonalization. (20%)

(4) Let a force vector $\mathbf{F}=(yze^{xyz}-4x)\mathbf{i}+(xze^{xyz}+z)\mathbf{j}+(xye^{xyz}+y)\mathbf{k}$ for all x, y and z. Determine whether **F** is conservative. If it is, find a potential function for **F**.

(20%)

(5) Find the steady-state temperature $u(r)$ in the circular cylinder shown in Fig.2 where $u=0$ at $z=0$, $u=0$ at $r=2$, $u=u_0$ at $z=4$.

[hint: (a) Laplacian in cylindrical coordinates is given as $\nabla^2=\dfrac{\partial^2}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2}{\partial\vartheta^2}+\dfrac{\partial^2}{\partial z^2}$;

(b) $(d/dt)[t\,J_1(t)]=t\,J_0(t)$]

(20%)

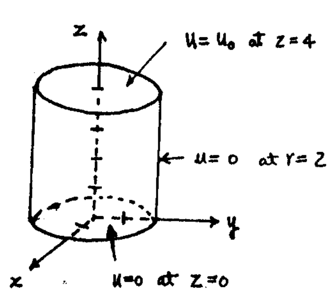

Fig. 2