

1. Given a solution of the following differential equation

$$(1+x)x^2y'' - (1+2x)xy' + (1+2x)y = 0, \quad (20)$$

as $y_1(x) = x$, find the second solution $y_2(x)$.

2. Given a matrix A as follows

$$A = \begin{pmatrix} 7.3 & 0.2 & -3.7 \\ -11.5 & 1.0 & 5.5 \\ 17.7 & 1.8 & -9.3 \end{pmatrix}, \quad (20)$$

find the eigenvalues of A^6

3. Given a function $f = xyz$, of which the derivation at point $P(x, y, z)$ in the direction $\vec{a} = \vec{i} - 2\vec{j} + 2\vec{k}$ is $7/3$, in the direction $\vec{b} = 2\vec{i} + \vec{j} + 2\vec{k}$ is $1/3$ and in the direction $\vec{c} = -2\vec{i} + 2\vec{j} - \vec{k}$ is $-11/3$, find the point $P(x, y, z)$.

(20)

4. Given $w = 3x^2y - y^2 + y^3$, evaluate the following integral

$$I = \oint_C \frac{\partial w}{\partial n} ds, \quad (20)$$

where C is the elliptic curve $25x^2 + y^2 = 25$, and n denotes the outer normal direction of C .

5. Determine the general solution of the following boundary value problem :

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= 0, & 0 < x < a, & \quad 0 < y < \infty \\ \frac{\partial u}{\partial x} &= 0, & x = 0, & \quad 0 \leq y \leq \infty \\ \frac{\partial u}{\partial x} &= 0, & x = a, & \quad 0 \leq y \leq \infty \\ u &= \frac{x}{a}, & y = 0, & \quad 0 \leq x \leq a \\ u &= 0, & y = \infty, & \quad 0 \leq x \leq a \end{aligned} \quad (20)$$