(1) Show that the given function ϕ is a solution of the differential equation for any choices of the constants c_1 and c_2.

$$x^2\frac{d^2y(x)}{dx^2}+2x\frac{dy(x)}{dx}-y(x)=0;\quad \phi(x)=\frac{1}{\sqrt{x}}\left(c_1x^{\sqrt{5}/2}+c_2x^{-\sqrt{5}/2}\right)\text{ for } x>0. \qquad (15\%)$$

(2) Find a solution of $\dfrac{d^2y(x)}{dx^2}+4y(x)=\tan(2x)$ on the interval $(-\pi/4,\pi/4)$. (15%)

(3) Let $\mathbf{R}(t)=r\cos(\omega t)\,\mathbf{i}+r\sin(\omega t)\,\mathbf{j}$ be the position vector of a particle moving in the xy-plane.

(a) Show that the angular speed is ω. (7%)

(b) Determine the acceleration vector $\mathbf{a}(t)$. (8%)

(4) Evaluate $\oint_c [2x\cos(2y)\,dx-2x^2\sin(2y)\,dy]$ for every positively oriented piecewise-smooth simple closed curve in the plane. (15%)

(5) Consider a simply supported beam subject to a uniformly distributed load ($q(x)=q_0$). The length, the bending rigidity and the flexural deflection of the beam are denoted as L, EI and y, respectively. The governing equation and the boundary conditions are given by

ODE: $EI\dfrac{d^4y(x)}{dx^4}=q_0$,

BC: $y(0)=y''(0)=0$,

$y(L)=y''(L)=0$.

Solve for the flexural deflection by using the Fourier sine half-range expansion method. (20%)

(6) Find the steady-state temperature distribution for a plate extending over the right quarter-plane $x\ge 0$, $y\ge 0$. The governing equation and associated boundary conditions are given as

PDE: $\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0 \qquad (x>0,\quad y>0)$,

BC: $u(0,y)=0 \qquad (0<y<\infty)$,

$$u(x,0)=\begin{cases}4 & 0\le x\le 2,\\ 0 & 2<x\le\infty.\end{cases} \qquad (20\%)$$