1. Solve the following differential equations

   a.
   $$(1 - x^2)y'' - 2xy' + 12y = 0, \tag{15}$$

   b.
   $$y'' - y = -2\sin t + \delta(t - 1), \qquad y(0) = 0, \qquad y'(0) = 2, \tag{15}$$

   where $\delta(.)$ denotes the Dirac delta function.

2. Is the quadratic form $\mathbf{x}^T \mathbf{A} \mathbf{x}$ always positive ? why ?
   Where
   $$\mathbf{A} = \begin{pmatrix} 32 & -24 & -8 \\ 16 & -11 & -4 \\ 72 & -57 & -18 \end{pmatrix}, \tag{15}$$

   and $\mathbf{x}^T = \{x_1, x_2, x_3\}$

3. Prove that the eigenvectors of a symmetric matrix corresponding to different eigenvalues are orthogonal.

   $$(15)$$

4. For a temperature distribution $T(x, y, z) = x^2 z + yz^2$ in a cone represented by the position vector as
   $$\vec{r} = u\cos v \vec{i} + u\sin v \vec{j} + 2u\vec{k}, \tag{20}$$

   find dT/dn at position $P(1, 0, 2)$ in the outer normal direction $\vec{n}$.

5. Solve the following partial differential equation.
   $$\nabla^2 u(r, \theta) = 0, \qquad (0 < r < 1, 0 < \theta < 2\pi)$$
   $$u(r, 0) = u(r, 2\pi) = 0$$
   $$u(1, \theta) = \cos(2\theta)$$

   $$(20)$$