1. The governing equation for the deflection of a rotating shaft subject to its own weight is given by

$$EI\frac{d^4y}{dx^4} - \rho A\Omega^2 y = \rho Ag, \quad (E, I, \rho, A, g, \Omega \text{ are constants}). \qquad (20)$$

Find the general solution and also the solution subject to the boundary condition

$$y = 0, \qquad dy/dx = 0 \qquad \text{at } x = \pm l.$$

2. Use the Laplace transform method to solve the simultaneous differential equations

$$4\frac{d^2u}{dt^2} + \frac{d^2v}{dt^2} - v = 0, \qquad \frac{d^2u}{dt^2} - u - v = 0, \qquad (20)$$

where $u(0) = 1$ and $u'(0) = v(0) = v'(0) = 0$.

3. If $\psi$ is any scalar field apply the divergence theorem to $\mathbf{a}\psi$, where $\mathbf{a}$ is any constant vector, and so deduce that

$$\iint_S \psi n dS = \iiint_R grad\,\psi dV, \qquad (20)$$

where $R$ is any region, $S$ is its boundary and $n$ is the unit outward normal to $S$, also, show that

$$\iint_S n dS = 0.$$

4. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & -3 & \sqrt{2} \\ -3 & 1 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 4 \end{bmatrix}. \qquad (20)$$

Verify that the eigenvectors of this matrix are mutually orthogonal.

5. A liquid diffuses through a porous membrane of thickness $a$. If the concentration $C(x, t)$ is maintained at $c_0$ on the $x = 0$ side of the membrane and $c_1$ on the $x = a$ side of the membrane, what is the concentration in the membrane after any transient terms have died away? The one-dimensional diffusion equation is

$$\frac{\partial^2 C}{\partial x^2} = \frac{1}{\kappa}\frac{\partial C}{\partial t}, \qquad (20)$$

where $\kappa$ is the constant diffusion coefficient.