

1. Solve the following simultaneous differential equations, where $x = x(t)$ and $y = y(t)$.

$$\begin{aligned} 4 \frac{d^2 x}{dt^2} &= -2x + y \\ 3 \frac{d^2 y}{dt^2} &= 2x - 2y \end{aligned} \quad (15)$$

2. Solve the following integral equation by Laplace transform method.

$$y(t) = e^{-t} - 2 \int_0^t \cos(t-u)y(u)du, \quad (15)$$

3. Is the following matrix A a positive definite one? Why?

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}, \quad (15)$$

4. Find the directional derivative of $f(x, y) = x^4 - 3x^3y + x^2y^2$ at $(2, 1)$ along the curve $x = t^2 + 1$, $y = t^3$ in the direction of increasing t .

(15)

5. Verify Green's theorem by the given vector

$$\vec{F} = 3y\vec{i} - 2xy\vec{j},$$

along the circle C

$$(x-3)^2 + (y-2)^2 = 16, \quad (15)$$

6. Solve the following partial differential equation.

$$\nabla^2 u(x, y) = 0, \quad 0 < x < a, \quad 0 < y < b$$

$$u(0, y) = 0, \quad u(a, y) = y$$

$$u(x, 0) = 0, \quad u(x, b) = x$$

(25)