1. *Building Code Requirements for Portland Cement Concrete* specifies that strength tests must meet the following two requirements:

   (1) The average ($\bar{x}$) of any three consecutive strength tests shall be equal to, or greater than the specified strength, f_c', i.e., $\bar{x} \geq f_c'$.

   (2) No individual strength test (x) shall be more than 35 kg/cm^2 below the specified strength, f_c', i.e., $x \geq f_c' - 35$.

   In order to minimize the probability of occurrence of strengths below f_c' to be l%, an average compression strength (f_{cr}') needs to be determined.　Please calculate the f_{cr}' value when $f_c' = 210$ kg/cm^2 and standard deviation $\sigma = 40$ kg/cm^2. (15 points)

   ## Values Provided for Your Calculations

Student t Distribution				Cumulative Normal Probability Distribution			
D.F.	Upper-Tail Area α			z	0.4	0.8	1
	0.1	0.05	0.025	$\Phi(z)$	0.65	0.79	0.84
9	1.383	1.833	2.262	z	1.28	1.64	2.33
8	1.397	1.860	2.306	$\Phi(z)$	0.9	0.95	0.99

2. (1) A regression model is as follows: $Y = b \cdot X$.　Please use the least square method to derive the expression for regression coefficient b. (10 points)

   (2) A regression model of predicting strength from sample's dimension is found to be as follows:

   $$\text{Strength (pound/inch}^2\text{)} = 36.29 \cdot \text{Dimension (inch)}$$

   If the model is in a metric system, the equation becomes as follows:

   $$\text{Strength (kg/cm}^2\text{)} = b \cdot \text{Dimension (cm)}.$$

   Please determine the b value. (5 points)

   Note: 1 inch = 2.54 cm, 1 pound = 0.45 kg

3. One type of additive is claimed to have the effect of increasing the strength of a material more than 10 MPa.　Nine sets of samples were prepared and tested.　You recorded results <u>with and without</u> additive and obtained following data:

With (MPa)	103	110	85	93	101	96	82	86	89
Without (MPa)	86	92	82	86	90	93	83	80	76

   (1) Do the test results support the "more than 10 MPa" claim at a level of significance of 5%?　Please write down the testing hypothesis and show your calculations. (10 points)

   (2) What is the strength increase you estimate with 90% confidence interval? (5 points)

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4. A contractor is building a project that includes five kilometers of asphalt pavements. Assume that he will follow the same rolling pattern throughout the project. After considerable testing on the first two kilometers of pavement, the contractor claims that he is achieving an in-place density of 2.31 kg/cm with a standard deviation of 0.025 kg/cm^3. At the beginning of the third kilometer a highway agency inspector requests that a control strip be identified. A total of 30 nuclear density readings is obtained from the control strip at random locations.

    (1) What is the probability that the mean of these 30 test results will be between 2.30 kg/cm^3 and 2.32 kg/cm^3? (5 points)

    (2) If the mean of the 30 test results were equal to 2.29 kg/cm^3, how likely is it that the contractor's claims regarding the process mean and standard deviation are correct? (10 points)

5. Let X_1, X_2, ..., X_n be a random sample of size n from a normal distribution $N(\mu, \sigma^2)$. Consider the following point estimators of μ: (1) $\hat{\mu}_1 = \frac{1}{n}\sum_{i=1}^{n} x_i$, (2) $\hat{\mu}_2 = x_1$, and (3)

    $$\hat{\mu}_3 = \frac{x_1}{2} + \frac{1}{2(n-1)}(x_2 + x_3 + ... + x_n)$$

    (1) Which of these estimators is unbiased? (5 points)

    (2) Which of these estimators is consistent? (5 points)

    (3) Find the relative efficiencies: $Var(\hat{\mu}_1)/ Var(\hat{\mu}_2)$, $Var(\hat{\mu}_1)/ Var(\hat{\mu}_3)$, $Var(\hat{\mu}_2)/ Var(\hat{\mu}_3)$ (5 points)

6. Please decide the best answer for each question. (25 points)

    (1) Different hypotheses for statistical tests are presented as follows. What are the appropriate hypotheses? (a) $H_0: \mu \neq \mu_0$ vs. $H_1: \mu = \mu_0$ (b) $H_0: \mu > \mu_0$ vs. $H_1: \mu \leq \mu_0$ (c) $H_0: \mu < \mu_0$ vs. $H_1: \mu \geq \mu_0$ (d) $H_0: \mu \geq \mu_0$ vs. $H_1: \mu < \mu_0$ (e) $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$

    (2) One researcher would like to determine the effect of manpower A, B and C on the timing of finishing a project. Three types of manpower were tested. By running one-way ANOVA, she obtained the following results: SSR =300, MSE = 20, SSE = 120. You are asked to determine the degree of freedom, $v_1 = $ __(a)__, $v_2 = $ __(b)__, and F value = __(c)__. Based upon the statistic, you may __(d)__ (reject or accept) the null hypothesis that the manpower has no influence on the timing of finishing a project. (Table F value = 5.14)