1. Consider the Sturm-Liouville problem

$$x^4\frac{d^2y}{dx^2}+2x^3\frac{dy}{dx}=\lambda y$$

$y(1)=0,\ y(2)=0.$

Find the eigenvalues and eigenfuctions.(Hint: $s = 1/x$)(15%)

2. Use the transformation $z = \sin(x)$ to solve

$$\frac{d^2y}{dx^2}+(\tan x)\frac{dy}{dx}+(\cos^2 x)y=0 \quad (10\%)$$

3. If F(s) and G(s) are the Laplace transform of f(t) and g(t),respectively. Express g(t) as an integral when

$G(s)=F(s)/(s+a)^2$ (15%)

4. Obtain a power series particular solution valid near x=0 for

$$x^2\frac{d^2y}{dx^2}+y=e^x/\sqrt{x} \quad (15\%)$$

5. Find the general solution of the equation

$$\frac{d\mathbf{y}}{dt}=\begin{pmatrix}7 & 4 & 4\\ -6 & -4 & -7\\ -2 & -1 & 2\end{pmatrix}\mathbf{y}$$

where $\mathbf{y}=(y_1, y_2, y_3)^T$.(15%)

6. Evaluate using residues the integral

$$\int_0^\infty \frac{dx}{1+x^3} \quad (15\%)$$

7.Show that the nonlinear (or quasi-linear) system of differential equations for u(x, y) and v(x, y)

$$u^n\frac{\partial u}{\partial x}+v^n\frac{\partial v}{\partial y}=0$$

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=0$$

with $n\geq 0$ can be transformed into a linear system (with variable coefficients) if x and y are considered functions of u and v. Derive the differential equations for x(u, v) and y(u, v). (15%)

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