1. Find a particular solution to the equation

$$\frac{d^2y}{dx^2}+y=x\sin x \quad (15\%)$$

2. Let f and g be defined, respectively, by

$f(x)=x^2, x\geq 0$ and $f(x)=0, x<0$

$g(x)=0, x\geq 0$ and $g(x)=x^2, x<0$.

Discuss the dependence of f and g (linearly dependent or independent).(10%)

3. Use the transformation $X=x/\sqrt{t}$ and t = t to express the equation

$$\frac{\partial^2 T}{\partial x^2}=\frac{\partial T}{\partial t}$$

(here T=T(x, t)) in terms of variables X and t through the following procedures. First

a) Compute $\partial T/\partial x$ and $\partial T/\partial t$ where T=T(X, t), and then (8%)

b) Compute $\partial T^2/\partial x^2$. (4%)

c) Use a) and b) to derive differential equation for T=T(X, t) (3%)

4.Solve the Laplace's equation

$$\nabla^2 u(x, y)=0$$

inside an elliptical region $(x/a)^2+(y/b)^2\leq 1$ subject to the boundary condition

$$u=1$$

on the whole elliptical boundary. (15%)

5. Bessel's equation is:

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-p^2)y=0$$

Determine the nature of the singularity at x = 0, 1, and ∞.(15%)

6.Compute the ranks of **A**, $\mathbf{A}^2$, $\mathbf{A}^3$, and $\mathbf{A}^4$, where

$$\mathbf{A}=\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{pmatrix} \quad (15\%)$$

7.Evaluate using complex variables

$$\int_0^\infty \frac{\sin x}{x}dx \quad (15\%)$$

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