

1. Given a homogeneous solution of the following differential equation

$$x^2 y'' + xy' + (x^2 - 1/4)y = x^{3/2}, \quad (15)$$

as $y_1 = \sin x/\sqrt{x}$, find the particular solution.

2. Find the solution of the differential equation $y'' + cy' + y = r(t)$, with $c > 0$ and $r(t)$ given as

$$r(t) = \frac{t}{12}(\pi^2 - t^2) \quad \text{if} \quad -\pi < t < \pi \quad \text{and} \quad r(t + 2\pi) = r(t), \quad (20)$$

3. Given the eigenvalues of a matrix

$$A = \begin{pmatrix} 4 & 3 & 9 & 9 \\ -8 & 3 & 5 & -4 \\ -8 & 0 & -2 & -8 \\ -16 & 6 & 14 & -5 \end{pmatrix}, \quad (15)$$

as $\lambda_1 = -0.2776 + 18.6896i$, $\lambda_2 = -0.2776 - 18.6896i$, $\lambda_3 = -0.1042$, and $\lambda_4 = 0.6593$, where $i = \sqrt{-1}$.

Find the eigenvalues of A^{-1} .

4. Calculate the work done by a force

$$\vec{F} = x^2 \vec{i} - xy \vec{j}, \quad (15)$$

from point (1,0) to (-1,0) along a curve of $x^2 + y^2/4 = 1$ in the upper plane (i.e. $y \geq 0$).

5. Given a velocity field as

$$\vec{v} = 7x \vec{i} - z \vec{k}$$

find the surface integral

$$I = \iint_S \vec{v} \cdot \vec{n} dA, \quad (15)$$

where \vec{n} is a unit outer normal vector for a sphere $x^2 + y^2 + z^2 = 4$

6. Solve the following partial differential equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 1, \quad (20)$$

with boundary conditions $\phi(0, y) = \phi(a, y) = \phi(x, 0) = \phi(x, b) = 0$.