1. Solve the following differential equation

$$ydx + (x - \ln y)dy = 0 \qquad (15)$$

2. The following differential equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \lambda y = 0$$

exists on the interval $-1 \le x \le 1$ and λ is a real eigenvalue. Is it always true for $y_i(x) \ne y_j(x)$ that $\int_{-1}^{+1} y_i(x) y_j(x) dx = 0$? Why ? (20)

3. Find the position vector $\vec{r}$ of a plane tangent to a surface $x^2 + y^2 + 4z^2 = 4$ at the point $(\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}})$. (15)

4. Prove that the work done by a gravitational force $\vec{F}(\vec{r})$ is independent of the path C. That is, the integral

$$W = \int_C \vec{F}(\vec{r}) \bullet d\vec{r} \qquad (15)$$

depends only on end points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$. Where $\vec{F}(\vec{r}) = -\nabla\phi(\vec{r})$, $\phi(\vec{r})$ is a scalar function, $\vec{r}$ is the position vector, and ∇ is the nabla.

5. Let $u_1(t)$ be a solution of the following equation

$\frac{d^2u_1(t)}{dt^2} + \frac{du_1(t)}{dt} + u_1(t) = \delta(t)$ with initial conditions $\frac{du_1(0)}{dt} = 0$ and $u_1(0) = 0$,

where a and b are constants and $\delta(t)$ is the Dirac delta function.

Assume $u_2(t)$ be a solution of the following equation

$\frac{d^2u_2(t)}{dt^2} + \frac{du_2(t)}{dt} + u_2(t) = f(t)$ with the same conditions $\frac{du_2(0)}{dt} = 0$ and $u_2(0) = 0$.

Prove that

$$u_2(t) = \int_0^t f(\tau) u_1(t-\tau) d\tau \qquad (15)$$

（背面仍有題目,請繼續作答）

6. Solve the following wave equaton

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}, \quad (0 < x, 0 < t)$$ (20)

with the following conditions

$y(x,0) = 0$, $(0 < x)$

$\frac{\partial y(x,0)}{\partial t} = e^{-2x}$, $(0 < x)$

$y(0,t) = \sin(3t)$, $(0 < t)$