1. Solve the following differential equation

$$(x+1)^2\frac{d^2y}{dx^2}-3(x+1)\frac{dy}{dx}+2y=0 \qquad (20)$$

with boundary conditions $y(0)=0$ and $\frac{dy(1)}{dx}=3$

2. Use Laplace transform to solve the deflection u(x) of a fixed-end beam of length l subjected to a concentrated loading P as shown in the following differential equation

$$EI\frac{d^4u}{dx^4}=P\delta(x-\frac{l}{3}), \qquad 0\le x\le l,$$

with the boundary conditions $u(0)=u(l)=0$ and $\frac{du(0)}{dx}=\frac{du(l)}{dx}=0$,

where $\delta(.)$ is the Dirac delta function and the rigidity EI and loding P are constant. (20)

3. Prove that

(a) The eigenvalues of a Hermitian matrix are always real. (10)

(b) The eigenvalues of similar matrices are the same. (10)

4. Calculate the following surface integral

$$I_s=\int_S \vec{F}\bullet\vec{n}dS,$$

where the vector field $\vec{F}=2z\vec{i}+(x-y-z)\vec{k}$,

$\vec{n}$ denotes the unit outer normal vector of the surface S: $z=x^2+y^2;\ x^2+y^2\le 6$, (15)

5. For a wave equation

$$\frac{\partial^2\phi}{\partial t^2}=\frac{\partial^2\phi}{\partial x^2},\quad 0\le t,\ 0\le x$$

(a) Show the D'Alembert's solution of the above equation (10)

(b) Solve $\phi(x,t)$ if $\phi(x,0)=\frac{d\phi(x,0)}{dt}=0$ and $\phi(0,t)=[u(t)-u(t-2)](-t^2+2t)$

where $u(.)$ denotes the unit step function. (15)

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