1. Find the amplitude of resonance for the vibration of a particle governed by

$$m\frac{d^2y}{dt^2}+c\frac{dy}{dt}+ky=P\cos\Omega t$$

where m, c, k, P and Ω are all constants. (20)

2. Solve the following differential equation

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=f(x),\quad y(0)=\frac{dy(0)}{dx}=0,\text{ where}$$

$f(x)=5x$, if $0<x<2$ and $f(x)=10$, if $2<x$, (20)

3. Prove that

(a) The eigenvalues of a Hermitian matrix are always real. (10)

(b) The eigenvalues of similar matrices are the same. (10)

4. Calculate the following surface integral

$$I_s=\int_S \vec{F}\bullet\vec{n}dS,$$

where the vector field $\vec{F}=2z\vec{i}+(x-y-z)\vec{k}$,

$\vec{n}$ denotes the unit outer normal vector of the surface S: $z=x^2+y^2$; $x^2+y^2\le 6$, (15)

5. For a wave equation

$$\frac{\partial^2\phi}{\partial t^2}=\frac{\partial^2\phi}{\partial x^2},\quad 0\le t,\ 0\le x$$

(a) Show the D'Alembert's solution of the above equation (10)

(b) Solve $\phi(x,t)$ if $\phi(x,0)=\frac{d\phi(x,0)}{dt}=0$ and $\phi(0,t)=[u(t)-u(t-2)](-t^2+2t)$

where $u(.)$ denotes the unit step function. (15)

G E06 3-1-3