

1.

If two loads are applied to a cantilever beam as shown in the figure below, the bending movement at 0 due to the load is $a_1X_1 + a_2X_2$



- (1) Suppose that X_1 and X_2 are independent random variables with means 2 and 4 kg respectively, and standard deviation 0.5 and 1 kg, respectively. If $a_1 = 5$ cm and $a_2 = 10$ cm, what is the expected bending movement and what is the standard deviation of the bending moment? (4 points)
- (2) If X_1 and X_2 are normally distributed, what is the probability that the bending moment will exceed 67 kg-cm? (4 points)
- (3) Suppose the positions of the two loads are random variables. Denoting them by A_1 and A_2 , assume that these variables have means of 5 and 10 cm, respectively, that each has a standard deviation of 0.5, and that all A_i 's and X_i 's are independent of one another. What is the expected moment now? What is the variance of the bending moment? (4 points)
- (4) If the situation is as described in part (1) except that $\text{Corr}(X_1, X_2) = -0.5$ (so that the two loads are not independent), what is the variance of the bending moment? (4 points)

Values Provided for Your Calculations

<i>Student t Distribution</i>				
DF	Upper-Tail Area α			
	0.01	0.005	0.05	0.025
19	2.539	2.861	1.734	2.093
20	2.528	2.845	1.729	2.086

<i>Normal Distribution</i>			
z	1.64	2.33	2.58
$\Phi(z)$	0.95	0.99	0.995

2.

A past study claims that college students in Taiwan spend an average of 18 hours per week on leisure activities. To assess whether or not the students at NCKU differ from the stated average, a random sample of 20 students at NCKU was selected and their time spend on leisure activities for a week was measured. The sample mean and standard deviation for this sample were 20.20 and 3.93 hours, respectively.

- (1) State the appropriate hypothesis. What test would you use and what assumptions do you need to make (if any)? Carry out the test at the significance level of 1% and state your conclusion. (4 points)
- (2) Describe Type I and II errors for this problem. (4 points)
- (3) All other things being equal, how would the probability of Type I error change if the sample size was 200? (2 points)
- (4) In which case would Type II error more likely: when the true mean $\mu=21$, or when $\mu=17$? Explain your answer. (2 points)
- (5) Find the 95% confidence interval for the mean _____ of hours per week students at NCKU spend on leisure activities. (4 points)

(背面仍有題目,請繼續作答)

G E06 3-2

3.

A cofferdam needs to be built to facilitate the reconstruction of a bridge abutment next summer. The random character of the maximum summer flood stage of the crossed stream, call it X , is defined by a triangular distribution with probability density function

$$f(x) = \begin{cases} 0 & x < 0 \\ x/50 & 0 \leq x \leq 10 \\ 0 & 10 < x \end{cases}$$

- (1) What is the cumulative function of X ? (4 points)
- (2) What is the mean of X ? (4 points)
- (3) What is the standard deviation of X ? (4 points)
- (4) What is the median of X ? (4 points)

4.

The top management in a large consulting company wanted the study the relationship between the salaries (Y , in thousands of dollars) of its consultants, and an index of the quality of advice (X_1), years of experience (X_2), and the amount (weekly) of consulting money (X_3) the company earned through the consultant. A multiple regression model was fitted based on the data obtained from a sample of 15 consultants. The fitted regression line turned out to be

$$\hat{Y} = 19.43 + 0.794X_1 + 0.370X_2 + 1.143X_3, \text{ SSE} = 38.57, \text{ SSTO} = 474.36.$$

- (1) Set up the analysis of variance (ANOVA) table. (10 points)
- (2) Calculate R^2 , the coefficient of multiple determination, and interpret it. (3 points)

5.

A joint probability density function $f(x, y) = cx^2y$ for $0 < x < 3$ and $0 < y < 2$.

- (1) Determine c . (4 points)
- (2) Find $P(X < 1, Y < 1)$ (4 points)

6

- (1) True or False (12 points)

- (a) Price is a discrete variable.
- (b) The mean is resistant to outliers.
- (c) Two non-null events that are independent cannot be mutually exclusive.
- (d) In a one-way analysis of variance with a categorical variable with three levels, three t tests to separately test null hypothesis $\mu_1 = \mu_2$, $\mu_1 = \mu_3$, $\mu_2 = \mu_3$ are equivalent to a single F test with null hypothesis $\mu_1 = \mu_2 = \mu_3$.
- (e) When comparing different multiple regression models, one should always prefer the model with the smallest residual sum of squares.

(f) In multiple regression, the fit of seven points (X_i, Y_i) with a degree of six polynomial as follows $Y = b_0 + b_1X + \dots + b_6X^6$ will have an $R^2 = 1$.

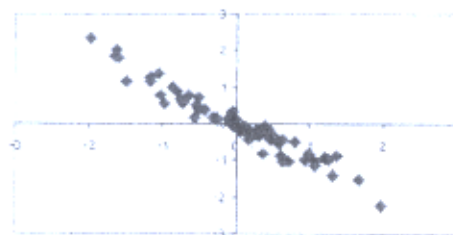
(2) An engineer concludes that a difference in sample is statistically significant at the 1% level.

Consider the statements: (a) The p-value for H_0 is more than 1%. (b) The difference would also be statistically significant at the 5% level. (c) if there were no difference in the population means, the chance of getting such a difference in the sample mean is 1% or less. (d) Statistical significance does not necessarily imply practical significance. What are the correct statements? (4 points)

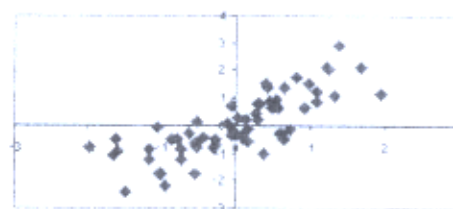
(3) Here are scatterplots for 4 data sets as shown below. Match the data sets to all statements which apply (matches can overlap, can be reused, and may not exist). (15 points)

- (a) The variables are strongly associated with each other.
- (b) The variables are moderately associated with each other.
- (c) The variables are not associated with each other.
- (d) The correlation is approximately $r = -0.95$.
- (e) The correlation is approximately $r = 0$.
- (f) The correlation is approximately $r = 0.8$.
- (g) The correlation is approximately $r = 0.95$.
- (h) There are no obvious outliers.
- (i) There is one serious outliers.
- (j) There are two probably outliers.
- (k) There are many outliers.
- (l) There is a positive linear relationship.
- (m) There is a negative linear relationship.
- (n) There is a curved relationship.
- (o) There is no apparent relationship.

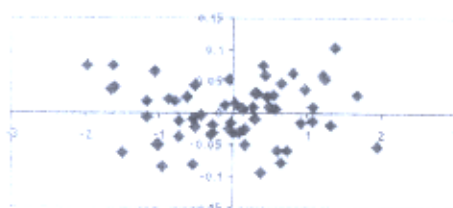
Data Set 1



Data Set 2



Data Set 3



Data Set 4

