1. Solve the following differential equation

$$x^3 \frac{d^3 y}{dx^3} - 3x^2 \frac{d^2 y}{dx^2} + 6x \frac{dy}{dx} - 6y = x^4 \ln x$$ (20)

2. Given the differential equations as follows

$$(1-x^2)\frac{d^2 y_1}{dx^2} - 2x\frac{dy_1}{dx} + ay_1 = 0 \text{ and}$$

$$(1-x^2)\frac{d^2 y_2}{dx^2} - 2x\frac{dy_2}{dx} + by_2 = 0$$ (20)

where a and b are constants and $a \neq b$.

Is $\int_{-1}^{1} y_1(x) y_2(x) dx = 0$ always true? Why?

3. Define the multiplication of matrices as follows

$$Q = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 3 & -2 & 0 \\ -2 & 5 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$, where x_1, x_2 and x_3 are any arbitrary real numbers.

Is $Q > 0$ always true ? why ? (20)

4. Verify the Green's theorem for the given vector $\vec{F} = xy\vec{i} + 2x\vec{j}$ along a square contour C with vertices at (0,0), (1,0), (1,1) and (0,1) (20)

5. Solve the following wave equation for a string of length L

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2} + \delta(x-a)e^{-i\omega t}$$, with the boundary conditions $y(0,t) = y(L,t) = 0$,

where $\delta(.)$ is the Dirac's delta function, $0 < a < L$ and ω is a constant. (20)