1. Given a homogeneous solution of the following differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-\frac{1}{4})y=0 \qquad (20)$$

as $y_1(x)=\dfrac{\sin x}{\sqrt{x}}$, find the other solution.

2. Given the differential equations as follows

$$(1-x^2)\frac{d^2y_1}{dx^2}-2x\frac{dy_1}{dx}+ay_1=0 \quad \text{and}$$

$$(1-x^2)\frac{d^2y_2}{dx^2}-2x\frac{dy_2}{dx}+by_2=0 \qquad (20)$$

where a and b are constants and $a\neq b$.

Is $\int_{-1}^{1} y_1(x)y_2(x)dx=0$ always true? Why?

3. Define the multiplication of matrices as follows

$$Q=[x_1 \quad x_2 \quad x_3]\begin{bmatrix}3 & -2 & 0\\ -2 & 5 & -3\\ 0 & -3 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$, where x_1, x_2 and x_3 are any arbitrary real numbers.

Is $Q>0$ always true ? why ? (20)

4. Verify the Green's theorem for the given vector $\vec{F}=3e^x y\vec{j}$ along a triangle contour C with vertices at (1,1), (2,3) and (1,6) (20)

5. Solve the following wave equation for a string of length L

$$\frac{\partial^2 y}{\partial x^2}=\frac{\partial^2 y}{\partial t^2}+\delta(x-a)e^{-i\omega t}$$, with the boundary conditions $y(0,t)=y(L,t)=0$,

where $\delta(.)$ is the Dirac's delta function, $0<a<L$ and ω is a constant. (20)