1. (i) (10%)

$$\text{(A) } \mathbf{F} = -\nabla\varphi, \quad \text{(B) } \oint \mathbf{F}\cdot d\mathbf{r} = 0, \quad \text{(C) } \nabla \times \mathbf{F} = 0.$$

In the following choices which one (a,b,c,d,e,f) is correct? Explain why.

(a) $(A) \Leftrightarrow (B) \Leftrightarrow (C)$,
(b) $(A) \Leftrightarrow (B) \not\Leftrightarrow (C)$,
(c) $(A) \Leftrightarrow (C) \not\Leftrightarrow (B)$,
(d) $(A) \not\Leftrightarrow (B) \Leftrightarrow (C)$,
(e) $(A) \not\Leftrightarrow (B) \not\Leftrightarrow (C)$.

(ii) (10%) Evaluate the integral

$$\oint_C (y - \sin x)\, dx + \cos x\ dy,$$

where C is the triangle from $(0,0) \to (\pi/2, 0) \to (\pi/2, 1) \to (0,0)$.

2. (i) (10%)Given two $n \times n$ matrices $\mathbf{A}$ and $\mathbf{B}$, suppose that there exists an invertible $n \times n$ matrix $\mathbf{C}$ such that

$$\mathbf{B} = \mathbf{C}^{-1}\mathbf{A}\mathbf{C}.$$

Show that $\mathbf{A}$ and $\mathbf{B}$ have the same eigenvelues.

(ii) (10%) Given the quadrative form

$$\mathbf{x}^T\mathbf{A}\mathbf{x} = d,$$

in which $\mathbf{A}$ is an $n \times n$ symmetric matrix, $\mathbf{x}$ is an $n \times 1$ matrix and d is a scalar. What are the conditions for the matrix $\mathbf{A}$ so that the value d is always positive for any given $\mathbf{x}$?

(背面仍有題目，請繼續作答)

3. (i) (10%) Find the solution of

$$\int_0^{2\pi} \frac{dx}{1-2p\cos x+p^2}, \quad 0<p<1.$$

(ii) (10%) Find the image of the circle $|z|=2$ after the mapping

$$w=z+\frac{1}{z},$$

where $z=x+iy$ and $w=u+iv$.

4. (i) (10%) Setting

$$\varphi(x.y)=e^{-(ax+by)}U(x.y),$$

transform the partial differential equation

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+2a\frac{\partial\varphi}{\partial x}+2b\frac{\partial\varphi}{\partial y}=0$$

into a different partial differential equation with unknown $U(x.y)$.
(ii) (10%) Find the general solution of $z(x,y)$

$$\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=0.$$

5. (i) (15%) Solve the differential equation by the method of Laplace transform

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+2y=t, \quad y(0)=\frac{dy(0)}{dt}=1.$$

(ii) (5%) What are Legendre polynomials?