編號：G 134 系所：土木工程學系丁組 科目：工程數學

本試題是否可以使用計算機： ☑可使用 ， ☐不可使用 （請命題老師勾選）

1. The Bessel function of the first kind is as follows

$$J_\nu(x)=\sum_{m=0}^{\infty}\frac{(-1)^m (x/2)^{2m+\nu}}{m!\Gamma(m+\nu+1)}$$, where $\Gamma(.)$ is the gamma function.

Prove

(a) $\dfrac{d(x^{-\nu}J_\nu(x))}{dx}=-x^{-\nu}J_\nu(x)$, (10)

(b) $J_{-n}(x)=(-1)^n J_n(x)$, if n is an integer (10)

2. Solve the following differential equation (20)

$\dfrac{d^2y}{dx^2}+y=\delta(x-a)$ with conditions as follows

$y(0)=y(L)=0$, where a is a constant and $0<a<L$,

and $\delta(.)$ is the Dirac's delta function.

3. Given a matrix as follows (20)

$$A=\begin{bmatrix}6 & -4 & 0\\ -4 & 7 & -3\\ 0 & -3 & 3\end{bmatrix},$$ of which the eigen vectors are

$\begin{Bmatrix}1.0\\ 0.4205\\ x_1\end{Bmatrix}$, $\begin{Bmatrix}x_2\\ 1.0\\ 1.1985\end{Bmatrix}$ and $\begin{Bmatrix}2.1047\\ x_3\\ 1.0\end{Bmatrix}$, please find x_1, x_2 and x_3.

4. Calculate the following integral of a vector $\bar{F}=2xy\bar{i}+zy\bar{j}-e^z\bar{k}$ (20)

$$I=\int_C \bar{F}\bullet d\bar{l}$$

where $|d\bar{l}|$ is a line segment of line C described as follows

C: a parabola $y=x^2$, $z=0$, from $(0,0,0)$ to $(2,4,0)$ in the xy-plane.

5. Solve the following partial differential equation (20)

$\dfrac{\partial^2\phi}{\partial x^2}+\dfrac{\partial^2\phi}{\partial y^2}=\sin y$, with the following conditions

$$\phi(0,y)=\phi(1,y)=0,\ \ \phi(x,0)=\frac{\partial\phi(x,\pi/2)}{\partial y}=0$$