本試題是否可以使用計算機： ☑可使用 ， ☐不可使用 （請命題老師勾選）

1. The Bessel function of the first kind is as follows

$$J_\nu(x) = \sum_{m=0}^{\infty} \frac{(-1)^m (x/2)^{2m+\nu}}{m!\Gamma(m+\nu+1)}$$, where $\Gamma(.)$ is the gamma function.

Prove

(a) $\dfrac{d(x^{-\nu}J_\nu(x))}{dx} = -x^{-\nu}J_\nu(x)$, (10)

(b) $J_{-n}(x) = (-1)^n J_n(x)$, if n is an integer (10)

2. Solve the following integral equation (20)

$$f(x) = \sin 2x - 2\int_0^x (x-u)^2 f(u)du$$

3. Given a matrix as follows (20)

$$A = \begin{bmatrix} 6 & -4 & 0 \\ -4 & 7 & -3 \\ 0 & -3 & 3 \end{bmatrix},$$ and define $A^2 = AA$, $A^3 = AAA$ ...... and so on,

please calculate the result of A^{50} .

4. A vector field is (20)

$$\vec{V} = y\vec{i} + x\vec{j} + x^2\vec{k} \quad ,$$

and the surface is described as

S: $z = 1-(x^2+y^2)$, $0 \le z$,

calculate the following flux integral

$$I = \iint_S \vec{V} \bullet \vec{n} dA$$

where $\vec{n}$ is an outer unit normal vector on the surface.

5. Solve the following partial differential equation (20)

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \sin y$$, with the following conditions

$$\phi(0,y) = \phi(1,y) = 0,\ \phi(x,0) = \frac{\partial\phi(x,\pi/2)}{\partial y} = 0$$