

編號： 134 系所：土木工程學系丁組

科目：工程數學

本試題是否可以使用計算機：可使用，不可使用（請命題老師勾選）

1. Solve the differential equation $\frac{dy}{dx} = (x+y+1)^2$. [Hint: let $u = x+y+1$] (20)

2. (a) Explain Cauchy-Riemann equations.

(b) Give the real part $u(x, y) = x^2 - y^2$ of an analytic complex function $f(z) = u(x, y) + iv(x, y)$, find the imaginary part $v(x, y)$.

(c) Determine the derivative of $f(z)$. (20)

3. (a) Explain half-range Fourier series expansion.

(b) Expand the function $f(x) = x+1$, $0 < x < \pi$ in a Fourier series and in a Fourier sine series (half-range expansion). (20)

4. (a) Explain the directional derivative of a function.

(b) Find the directional derivative of the function $f(x, y) = x^2 + y^2$ at point $(3, 4)$ in the direction $2\mathbf{i} + \mathbf{j}$.

(c) Find the maximum directional derivative of the function $f(x, y) = x^2 + y^2$ at point $(3, 4)$. (20)

5. Calculate the double integration $\iint_R xy dx dy = ?$, where $R: \begin{cases} 0 < x+y < 2 \\ 0 < x-y < 2 \end{cases}$. [Hint: let $u = x+y$, $v = x-y$] (20)