編號： 127　系所：土木工程學系乙組　　科目：工程數學

本試題是否可以使用計算機： ☑可使用 ， ☐不可使用 （請命題老師勾選）

1. Solve the differential equation $\frac{dy}{dx}=\frac{2x+y}{2x+y+1}$. [Hint: let $u=2x+y$] (20)

2. (a) Explain Cauchy-Riemann equations.
   (b) Give the real part $u(x,y)=x^2-y^2$ of an analytic complex function $f(z)=u(x,y)+iv(x,y)$, find the imaginary part $v(x,y)$.
   (c) Determine the derivative of $f(z)$. (20)

3. (a) Explain half-range Fourier series expansion.
   (b) Expand the function $f(x)=x^2,\quad 0<x<\pi$ in a Fourier series and in a Fourier sine series (half-range expansion). (20)

4. (a) Explain the directional derivative of a function.
   (b) Find the directional derivative of the function $f(x,y)=x+y^2$ at point $(3,4)$ in the direction $2\mathbf{i}+\mathbf{j}$.
   (c) Find the maximum directional derivative of the function $f(x,y)=x+y^2$ at point $(3,4)$. (20)

5. Calculate the double integration $\iint_R xydxdy=?,$ where $R:\begin{cases}0<x+y<2\\0<x-y<2\end{cases}$. [Hint: let $u=x+y$, $v=x-y$] (20)