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1. Determine the nature of the singularity (if any) at $z=0$ for the following $f(z)$. Can you expand these functions in powers of z convergent in a punctured disk $0<|z|<R$. (25%)

(a) $\sin(1/z)$
(b) $(\sin z)/z$
(c) $(\sin z)/z^2$
(d) $1/\sin(1/z)$
(e) $z\sin(1/z)$

2. Are the following statements true or false? If it is false, explain the reason. (16%)

(a) If $u(x,y)$ is harmonic in D, then it is the real part of an analytic function $f(z)$ in D.
(b) The real and imaginary parts of a complex analytic function are harmonic.
(c) If two analytic functions have the same real part $u(x,y)$, then $f(z)=g(z)$ identically.
(d) If $f(z)=u(x,y)+iv(x,y)$ with $u(x,y), v(x,y)$ harmonic, then $f(z)$ is analytic.

3. Solve the following equation (15%)

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)u(x,y)=0$$

in a unit disk with $u=1+\theta$ on the boundary.

4. Let $\boldsymbol{F}(x,y,z)=(x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k})/r^n$, where $r=\sqrt{x^2+y^2+z^2}$ and n is a positive integer.

(a) Show that $div\boldsymbol{F}=(3-n)/r^n$ (4%)

(b) Evaluate the surface integral $\iint_S \boldsymbol{F}\cdot\boldsymbol{n}dS$ for $n=2$ where S is the

（背面仍有題目，請繼續作答）

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sphere $x^2+y^2+z^2=a^2$. Can you use the divergence theorem?(5%)

(c) Evaluate the surface integral $\iint_S \boldsymbol{F}\cdot\boldsymbol{n}dS$ for $n=3$ where S is the sphere $x^2+y^2+z^2=a^2$. Can you use the divergence theorem? (5%)

5. Determine all possible solutions for the following equation

$$\begin{bmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

where λ is any real number.(15%)

6. Bessel's equation is:

$$x^2y''+xy'+(x^2-p^2)y=0$$

Determine the nature of the singularity at $x=\infty$ by transforming the independent variable to $z=1/x$. (15%)