1. Consider the second-order homogeneous linear differential equation

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0.$$

a) Find the two linearly independent solutions f_1 and f_2 of this equation which are such that

$f_1(0)=1$ and $f_1'(0)=0$

and

$f_2(0)=0$ and $f_2'(0)=1$ (5%)

b) Express the solution

$3e^{x}+2e^{2x}$

as a linear combination of the two linearly independent solutions f_1 and f_2 defined in (a). (5%)

2. Consider the differential equation

$(4x+3y^2)dx+2xydy=0$

a) Show that this equation is not exact.(5%)

b) Find an integrating factor of the form x^n, where n is a positive integer.(5%)

c) Multiply the given equation through by the integrating factor found in (b) and solve the resulting exact equation. (5%)

3. The function f has at (1,-1) a directional derivative equal to $\sqrt{2}$ in the direction toward (3,1), and $\sqrt{10}$ in the direction toward (0,2).

（背面仍有題目,請繼續作答）

a) Find the value of $\partial f / \partial x$ and $\partial f / \partial y$ at (1,-1).(5%)

b) Determine the derivative of f at (1,-1) in the direction toward (2,3). (5%)

4. Find a unit tangent vector to the curve of intersection of the plane $y - z + 2 = 0$ and the cylinder $x^2 + y^2 = 4$ at the point (0,2,4) (10%)

5. Evaluate the line integral

$$\int_c \frac{-y\,dx + (x-1)\,dy}{(x-1)^2 + y^2}$$

where c is any piecewise smooth simple closed curve containing the point (1,0) in its interior. (15%)

6. Evaluate

$$\int_{-\infty}^{\infty} \frac{\sin x}{x(1+x^2)} dx$$

by complex variable methods. (15%)

7. Show that any function f(t) can be expressed as the sum of two component functions, one of which is even and the other odd. (10%)

8. An important property of the Laplace transform is the convolution theorem. State this theorem and prove it. (15%)