

系所組別： 土木工程學系甲、乙、丁組

考試科目： 工程數學

考試日期： 0307，節次： 3

※ 考生請注意：本試題 可 不可 使用計算機

1. Solve the following ODE.

(a) (5%)  $y' = -2x/y$

(b) (5%) Show that the following equation is exact and solve it.

$$3x^2 y dx + x^3 dy = 0$$

(c) (5%)  $y' = 2xy - x^2$ ,  $y(0) = 1$

(d) (5%)  $x^2 y'' - 3xy' + 4y = 3x^2$

2. (20%) Solve the following initial value problem for system.

$$y_1' = 2y_1 - y_2$$

$$y_2' = -y_1 + y_2$$

$$y_1(0) = 1, \quad y_2(0) = 0$$

3. Using Laplace transforms, solve the following problems.

(a) (10%)  $y'' + 4y = \delta(t-1)$ ,  $y(0) = 1$ ,  $y'(0) = 0$

(b) (10%)  $y = 4t^2 - 2 \int_0^t y(\tau)(t-\tau) d\tau$  (hint: using the convolution theorem)

4. Evaluate the following line integrals.

(a) (7%)  $\int_C xy^2 dx + x^2 y dy$ ,  $C: r = [\cos t, 2 \sin t]$ ,  $t$  from 0 to  $\pi/2$ .

(b) (7%)  $\int_C xy^2 dx + x^2 y dy$ ,  $C$ : the straight-line segment from (1, 0) to (0, 2).

(c) (6%) Show that the line integral  $\int_C xy^2 dx + x^2 y dy$  is path independent.

5. Evaluate the following contour integrals.

(a) (5%)  $\oint_C \frac{dz}{z-2i}$ ,  $C$ : the circle  $|z| = 1$ , counterclockwise.

(b) (5%)  $\oint_C \frac{dz}{z-2i}$ ,  $C$ : the circle  $|z| = 2$ , clockwise.

(c) (5%)  $\oint_C \frac{dz}{z-i}$ ,  $C$ : the circle  $|z| = 3$ , counterclockwise.

(d) (5%)  $\oint_C \frac{dz}{z^2+1}$ ,  $C$ : the circle  $|z| = 2$ , counterclockwise.