

系所組別 土木工程學系甲、乙、丁組

考試科目 工程數學

考試日期：0307，節次：3

※ 考生請注意：本試題 可 不可 使用計算機

(1)

- (a) Determine
- $a(x)$
- so that by a change of variable
- $y = a(x)u(x)$
- then the equation

$$y''(x) + 2xy'(x) + y(x) = 0$$

becomes

$$u''(x) + r(x)u(x) = 0.$$

What is $r(x)$? (15%)

- (b) Solve
- $xy''(x) - y'(x) - 4x^3y(x) = 0$
- by setting
- $x = t^{1/2}$
- (15%)

(2)

- (a) Evaluate the line integral

$$\int_c (x + y) dy$$

where c is the ellipse $(x/a)^2 + (y/b)^2 = 1$ clockwise. (10%)

- (b) Evaluate
- $\text{curl}(\mathbf{u} \text{ div } \mathbf{u})$
- where
- $\mathbf{u} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$
- . (10%)

(3)

- (a) Find the derivative of
- $f = xyz$
- at the point
- $(1, 3, 2)$
- in the direction of the vector
- $2\mathbf{i} - \mathbf{k}$
- . (10%)

- (b) What is the maximum possible directional derivative of
- $f = xyz$
- at the point
- $(1, 3, 2)$
- and what is its direction? (10%)

(背面仍有題目,請繼續作答)

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(4)

Bessel's equation of order zero is

$$x^2 y''(x) + xy'(x) + x^2 y(x) = 0$$

One of the solution is

$$J_0(x) = 1 - \frac{x^2}{4} + \frac{x^4}{64} - \dots$$

The second solution exists of the form

$$J_0(x) \ln x + Ax^2 + Bx^4 + \dots$$

Find the two coefficients A and B (15%)

(5)

(a) Find values of a, b and c so that

$$f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + cy^2)$$

is entire. (5%)

(b) Evaluate

$$\oint_{|z-2|=1} \frac{e^{1/z} \sin z}{(e^z - 1)(z - 2)} dz \quad (10\%)$$