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1. A construction site has two different types of materials used for bridge reinforcements. Let X = the amount (ton) of brand A on hand, and Y = the amount of brand B on hand. Suppose the joint pdf of X and Y is as follows: (4 points each)

$$f(x,y)=\begin{cases} kxy, & x\ge 0,\ y\ge 0,\ 20\le x+y\le 30 \\ 0, & \text{otherwise} \end{cases}$$

(1) What is k value?

(2) Are X and Y independent?

(3) Compute P(X+Y≤25).

(4) What is the expected total amount of materials on hand?

(5) Compute Cov(X, Y), and Corr(X, Y).

(6) What is the variance of the total amount of materials on hand.

2. The data on the shear strength (MPa) of a material adding four different additives are observed. An investigation was carried out to see if there are any differences among additives considered.

Additive	Sample size	Sum	Average	Note:
1	6	344.96	57.49333	$\sum\sum x_{ij}^2 = 83140.06$, $\sum x_{i\bullet}^2 = 498642.95$
2	6	347.71	57.95167	$(\bar{x}_{i\bullet}-\bar{x}_{j\bullet})\pm Q\cdot\sqrt{\frac{MSE}{J}}$, Q = 3.96.
3	6	357.32	59.55333	
4	6	362.03	60.33833	

(1) Construct an analysis of variance (ANOVA) table. (8 points)

(2) State your null hypothesis, and test the hypothesis using a 0.05 level of significance. (2 points)

(3) Construct a 95% confidence interval (CI) for $\theta_1 = \mu_1 - \mu_2$, and $\theta_2 = \mu_1 - \mu_4$. (4 points)

(4) Following (3), what conclusion would you make regarding the difference? Why? (2 points)

Values Provided for Your Calculations

z	-1.00	1.64	1.96	2	2.5	3	$t_{0.05,49}$	$t_{0.1,49}$	$t_{0.05,50}$	$F_{0.05,3,20}$	$F_{0.05,4,20}$	$F_{0.05,3,21}$
Φ(z)	0.159	0.95	0.975	0.977	0.994	0.998	1.676	1.299	2.009	3.10	2.87	3.07

3. At Anycity College, the weekly working time of adult men is normally distributed with mean of 45 hours and standard deviation of 10 hours, and the weekly working time of adult women is normally distributed with mean of 40 hours and standard deviation of 4 hours. Six women and four men are randomly selected. Assume that all their working hours are independent. What is the probability that their total work time exceeds 440 hours within a week? (4 points)

(背面仍有題目,請繼續作答)

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4. Suppose the relationship between working hour (x) and productivity (y) is described by the simple linear regression model with true regression line $y = 65 - 1.2 \cdot x$ and $\sigma = 8$. Determine the following values: (4 points each)
   (1) $P(Y > 49 \text{ when } x = 20)$
   (2) $P(Y_1 - Y_2 > 10.11 \text{ when } x_1 = 25, x_2 = 24)$
   (3) $P(43 \le Y \le 75 \text{ when } x = 5)$
   (4) What is the value of Y that is exceeded by 95% of the samples when x = 5?

5. A study was conducted to compare the effect of two construction methods on the performance of workers. This work carried out in a large construction site. A total of 100 workers with the same skill level were divided at random into two groups. One group is designed by Method A, and the other by Method B. At the end of the project, all of the workers were evaluated for their performance. Suppose the results of the performance are normally distributed with the following data. Do the results give evidence of a difference in the effectiveness of the construction methods at α =0.1? (4 points)

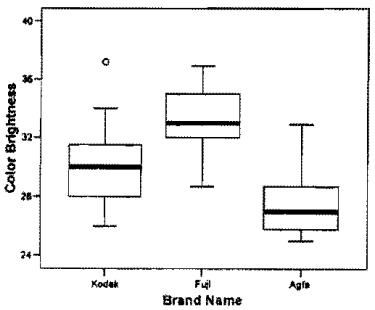
Group	$\bar{X}$	s	n
Method A	54	4	50
Method B	49	3	50

6. Answer the following questions. (4 points each)

(1) Suppose $X_1,\ldots,X_4$ are independent and identically distributed *(iid)* random variables with mean = 5 and variance = 2 and $Y_1,\ldots,Y_{10}$ are *iid* random variables with mean = 4 and variance = 6. Assume X_i and Y_j are independent for all $i = 1,\ldots,4$ and $j = 1,\ldots 10$. Evaluate the following. (a) $E(2+3X_1-2Y_1)$ (b) $Var(2+3X_1-2X_2)$ (c) $E\left(2+\frac{3}{4}\sum_{i=1}^{4}X_i-\frac{2}{10}\sum_{j=1}^{10}Y_j\right)$ (d) $Var\left(2+\frac{3}{4}\sum_{i=1}^{4}X_i-\frac{2}{10}\sum_{j=1}^{10}Y_j\right)$

Use the information in the following setting to answer questions 2 through 5.

It is desired to test whether the brightness of three different brands of film are equal. To that end, random samples from 15 of the ratings from each brand were taken, and the following boxplot was generated:



ANOVA

Color Brightness

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups			47.854	5.285	.009
Within Groups			9.054		
Total	475.970				

(2) Do you think the assumptions for the one-way ANOVA model are met? Why or why not? (a) No, because

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the means are very different from one another. (b) No, because the distributions look very heavily left skewed. (c) Yes, because the outlier wouldn't really have been an outlier if it had been in the Fuji film population. (d) Yes, because the interquartile ranges look approximately equal, so I think the variances are probably similar, and things look pretty normal. (e) This cannot be determined without studying the actual data values with a dot plot.

(3) What are the degrees of freedom for the numerator of this F-statistic?

(4) What is your conclusion about the three kinds of film, at the .01 level of significance? (a) The mean color brightness of at least one of the three groups is significantly different from the others. (b) We don't have evidence to say that any of the three kinds of film has a mean color brightness different from the other kinds of film. (c) The mean color brightness of the three kinds of film is about the same. (d) Since we're running a two-sided test about the means of the three groups, we divide the p-value by two, and conclude that the means are different. (e) Two of the above are true.

(5) What does the p-value of 0.009 mean in context of the problem? (a) The probability that we would conclude that the color brightness was different for the different brands of film when actually the color brightness was the same for all three brands is 0.009. (b) The probability of taking a random sample from three brands of film that had equal color brightness and getting sample means this different or even less different is 0.009. (c) The probability of taking a random sample from three brands of film that had equal color brightness and getting sample means this different or even more different is 0.009. (d) The probability that we would conclude that the mean color brightness was different for the three groups when in reality the color brightness is different for at least one of the groups is 0.009. (e) The probability that we would conclude that the mean color brightness was the same for all three groups when in reality it was different for at least one of the groups is 0.009.

(6) The number of accidents happening on a highway in a year is a Poisson distributed. Thus, the length of time between accidents follows: (a) Poisson distribution, (b) exponential distribution, (c) Weibull distribution, (d) normal distribution, (e) binomial distribution

(7) What is the wrong statement regarding the assumptions of linear regression analysis? x is denoted as an independent variable and y a dependent variable. (a) For each y value, the collection of all possible x values corresponding to that y value has a normal distribution. (b) The variance from the regression line of all the y values corresponding to a specific x value will be the same no matter what x value we choose.(c) The y values in the scatter diagram are unrelated. (d) x and y are linearly related.

(8) H_0: New design is safe. Which of the following is the type II error? (a) Approve an unsafe design, (b) Disapprove an unsafe design, (c) Disapprove a safe design, (d) Approve a safe design, (e) There is no type II error.

(9) What is the standard error for $2\bar{x}+1$? (Hint: let $X_1,\ldots,X_n$, the random sample of size n , from the distribution with the mean, μ and the variance σ^2 The answer will be in terms of the notation)