

系所組別： 工程科學系乙組

考試科目： 數值分析

考試日期： 0219 · 節次： 1

※ 考生請注意：本試題 可 不可 使用計算機

1. 30%

Use any method you like to prove the following relations: (h is the equal space distance between f_i).

$$(a) f'(x_0) = \frac{-f_2 + 4f_1 - 3f_0}{2h} + O(h^2) \quad (10\%)$$

$$(b) f''(x_0) = \frac{-f_3 + 4f_2 - 5f_1 + 2f_0}{h^2} + O(h^2) \quad (10\%)$$

$$(c) \text{ For non-equal spacing points, } f(x-s), f(x), f(x+t), \text{ find } f'(x) = ? \quad (10\%)$$

2. 20%

Given the following set of data:

x	1	2	3	4	5
$f(x)$	2.4	2.6	3.2	3.6	4.2

(a) Calculate $f'(3)$ and $f''(3) = ?$ (10%)

(b) Compute $\int_1^5 f(x)dx = ?$ (10%)

3. 30%

Suppose for $Ax=b$, that $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 3 & 4 \\ 2 & 4 & a \end{bmatrix}$

What is the smallest value of a for which convergence will be obtained by

(a) using Jacobi method? (15%)

(b) using Gauss-Seidel method? (15%)

4. 20%

We want to use the Newton's method to solve the following nonlinear equations:

$$\begin{cases} x_1^2 + x_2 - 37 = 0 \\ x_1 - x_2^2 - 5 = 0 \\ x_1 + x_2 + x_3 - 3 = 0 \end{cases}$$

Derive the iteration expression for finding the roots.