※ 考生請注意：本試題 $\square$ 可 区不可 使用計算機

1．$(20 \%)$ In the context of operating systems，one approach to prevent deadlocks between concurrent processes is to impose a total ordering of all resource types，and each process must request resources in an increasing order of enumeration．（1）What is a＂total ordering＂？（Please give formal definitions）（2）Why the total ordering approach can prevent deadlocks？（Please give a proof．）

2．（ $20 \%$ ）Find the finite－state automata equivalent to the nondeterministic finite－state automata defined as follows，where the input set of symbols $=\{a, b$ ， $\mathrm{c}\}$ ，the set of states $=\left\{\mathrm{S}_{0}, \mathrm{~S}_{1}, \mathrm{~S}_{2}\right\}$ ，and the initial state $=\left\{\mathrm{S}_{0}\right\}$ ．

| Input <br> ／states | a | b | c |
| :--- | :--- | :--- | :--- |
| $\mathrm{S}_{0}$ | $\left\{\mathrm{~S}_{1}\right\}$ | Empty set | Empty set |
| $\mathrm{S}_{1}$ | $\left\{\mathrm{~S}_{0}\right\}$ | $\left\{\mathrm{S}_{2}\right\}$ | $\left\{\mathrm{S}_{0}, \mathrm{~S}_{2}\right\}$ |
| $\mathrm{S}_{2}$ | $\left\{\mathrm{~S}_{0}, \mathrm{~S}_{1}, \mathrm{~S}_{2}\right\}$ | $\left\{\mathrm{S}_{0}\right\}$ | $\left\{\mathrm{S}_{0}\right\}$ |

3．$(15 \%)$ Let $c_{n}$ denote the number of strings over $\{0,1,2\}$ of length $n$ that contain an even number of 1 ＇s．（1）Write a recurrence relation and initial condition that define the sequence $c_{1}, c_{2}, \ldots$（2）Solve the recurrence relation to obtain an explicit formula for $c_{n}$ ．

4．$(10 \%)$ Consider the tic－tac－toe game．Assume that the first player marks an X at the center square．The other player is supposed to mark an O at some square． Draw a two－level game tree，with the root having an X in the center square． Omit symmetric positions．Evaluate all the vertices use the evaluation function： $\operatorname{ef}(\mathrm{P})=\mathrm{NX}-\mathrm{NO}$ ；where NX （respectively，NO）is the number of rows， columns，or diagonals containing an X （respectively， O ）might complete，if square $P$ is chosen．Which is the best position for $O$ to move？

5．Answer the following questions briefly：
（1）（5\％）Give examples of distinct binary trees $B_{I}$ and $B_{2}$ ，each with two vertices， with the preorder vertex listing of $B_{1}$ equal to the preoder listing of $B_{2}$ and the postorder vertex listing of $B_{I}$ equal to the postorder listing of $B_{2}$ ．
（2）（5\％）Show that any simple，connected graph with 31 edges and 12 vertices is not planar．
（3）$(5 \%)$ Prove that a tree is a bipartite graph．
（4）（5\％）Let $G$ be a connected graph．The distance between vertices $x$ and $y$ in $G$ ， $\operatorname{dist}(x, y)$ is the length of a shortest path from $x$ to $y$ ．The diameter of G is： $d(G)=\max \{\operatorname{dist}(x, y) \mid x$ and $y$ are vertices in $G$.
What is the diameter of an n－cube？In the context of parallel computing， what is the meaning of this value？
（5）（5\％）Let $\boldsymbol{A}$ be the adjacency matrix of a graph $\boldsymbol{G}$ with n vertices．Let $\boldsymbol{B}=\boldsymbol{A}^{l}+$ $A^{2}+\ldots+A^{n-1}$ ．If some off－diagonal entry in the matrix $\boldsymbol{B}$ is zero，what does this imply？
（6）（5\％）A marathon beginner decides to try to complete the local marathon．The beginner（runner）will stop if the marathon is completed or after three attempts．The probability of completing the marathon in one attempt is $1 / 3$ ． Analyze the following argument that，assuming independence，What is the probability that the beginner is able to complete the marathon？
（7）（5\％）Suppose that there are three persons who each randomly choose a box among 12 consecutive boxes．What is the probability that the three boxes are consecutive？

