系所組別：工程科學系乙、丙組
考試科目：計算機數學 考試日期：0223，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Prove the statement: If *S* is an arbitrary set, and *P(S)* is the collection of subsets of *S*, then the relation $\subseteq$ is in fact a partial ordering of *P(S)*. (*P(S)* is also called the 'power set' of *S*). (10%)

2. Consider the equation $x_1 + x_2 + x_3 = 5$, where x_1, x_2, and x_3 must all be nonnegative integers.
   (1)How many solutions are there? (Note: The solutions 2 + 2 + 1 = 5 and 1 + 2 + 2 = 5, for example, are considered to be distinct.)(5%)
   (2)More generally, how many distinct solutions are there to the equation $x_1 + x_2 +\ldots+ x_n = k$, where each x_i must be a nonnegative integer?(5%)

3. Consider the algebraic expression $(((7-5)*2) \div 2)+(6*(9 \div 3))$.
   (1)Build the corresponding binary tree. (Note that parenthesis will not be shown in the tree.) (5%)
   (2)Give the corresponding reverse Polish notation expression (or postorder traversal). (5%)

4. Suppose you had a supply of two-input AND gates.
   (1)How many of these would be needed to build a five-input AND gate? (2%)
   (2)How would you build a circuit that accepts all inputs? Rejects all inputs?(4%)
   (3)How would you 'mask' one-bit of input? (Note: you can use inverters). (4%)

5. Construct a NFA (nondeterministic finite state automaton) that accepts the set of strings of 0s and 1s containing no three 1s in a row. (10%)

6. (1) A famous basketball player is shooting free throws. He will make the shot with probability 0.90, and will miss with probability 0.10. Suppose he attempts 10 shots in a row. What is the probability that he misses at least one? (5%)
   (2)Let Ω = { 1, 2, 3, 4, 5, 6, 7, 8} be a uniform probability space, i.e., each of the eight atoms has probability 1/8. Let A = {1, 2, 3, 4}. Find all events B such that Pr {A|B} = Pr{A}. (5%)

（背面仍有題目，請繼續作答）

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7. (1)Show that the complete graph K_n is planar for $n = 1, 2, 3, 4$ but for no larger values of n. (5%)

(2)Consider a complete bipartite graph $K_{2,3}$, which is defined to be a bipartite graph with $|A|=2$, and $|B|=3$, with every A-vertex connected to every B-vertex. Draw the graph for $K_{2,3}$. How many complete matchings are there? (5%)

(3)Does either $K_{2,3}$ or K_5 have a Euler cycle, respectively? Why or why not?(5%)

(4)Does either $K_{2,3}$ or K_5 have a Hamiltonian cycle, respectively? Why or why not? (5%)

8. (1)Prove by induction that $5^n - 4n - 1$ is exactly divisible by 16 for $n = 1, 2, 3, \ldots$.(5%)

(2)Consider the statement $S(n)$: $n^2 - n + 41$ is prime (i.e. not divisible by any positive integer except itself and 1) for all $n = 0, 1, 2, \ldots$.

(a)Verify that $S(0)$, $S(1)$, $S(2)$ and $S(3)$ are true.(2%)

(b)Why must an attempt to prove $S(n)$ by induction fail?(Hint: You can show a counter example.) (3%)

(3)Consider the homogeneous difference equation with nonconstant coefficients $x_n = nx_{n-1}$, $n = 1, 2, 3, \ldots$, with initial conditions $x_0=1$. Find a general solution to x_n. (Hint: you could calculate enough terms to see a pattern, and confirm your guess using mathematical induction.) (10%)