

系所組別：工程科學系乙組

考試科目：數值分析

考試日期：0223，節次：1

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%)

We want to use the Newton's method to solve the following nonlinear equations:

$$\begin{cases} x_1^2 + x_2 - 37 = 0 \\ x_1 - x_2^2 - 5 = 0 \\ x_1 + x_2 + x_3 - 3 = 0 \end{cases}$$

Derive the iteration expression for finding the roots.

2. (20%)

The *van der Pol* equation is a model of an electronic circuit that arose back in the days of vacuum tubes:

$$\frac{d^2 y}{dt^2} - (1 - y^2) \frac{dy}{dt} + y = 0. \quad \text{Given the initial conditions, } y(0) = y'(0) = 1, \text{ solve this equation from } t=0 \text{ to } t=0.6 \text{ using any integration method with a step size of } 0.2.$$

3. (25%)

Given the following set of data:  $\frac{x}{f(x)} \left| \begin{array}{c|c|c|c} -0.1 & 0.0 & 0.2 \\ \hline -2.3 & -2.2 & -1.94 \end{array} \right|$

(a) Find the 2<sup>nd</sup> degree Lagrange interpolating polynomial passing through the above points. (10%)

(b) Find the 2<sup>nd</sup> degree polynomial via the least square approach. (10%)

(c) What is  $f(0.1)$  from (a) and (b)? (5%)

4. (20%)

We want to integrate  $\int_{-1}^1 f(x) dx$  numerically by the two-term Gaussian quadrature i.e.

$$\int_{-1}^1 f(x) dx = c_1 f(t_1) + c_2 f(t_2). \quad \text{Find } c_1, c_2, t_1 \text{ and } t_2.$$

5. (15%)

(a) For a value in a decimal (十進位) system is 11.1, what is its expression in a binary (二進位) system? (5%)

(b) Form part (a), how many bits does a computer require to store the binary digits for the decimal number 11.1? What will happen if a personal computer has only 32 bits to store the number? (10%)