

※ 考生請注意: 本試題不可使用計算機。請於答案卷(卡)作答, 於本試題紙上作答者, 不予計分。

1. Vector analysis is an essential mathematical tool in electromagnetics. (30%)

(1) What is the physical definition of the gradient of a scalar field? What is the physical definition of the divergence of a vector field? What is the physical definition of the curl of a vector field? (15%)

(2) Prove that the curl of the gradient of any scalar field is zero. Show that the divergence of the curl of any vector field is zero. (15%)

2. For a very long coaxial cable that has an inner conductor of radius r_i and an outer conductor of inner radius r_o , the space between the conductors is filled with two coaxial layers of dielectrics. The dielectric constants of the dielectrics are ϵ_1 for $r_i < r < a$ and ϵ_2 for $a < r < r_o$. (30%)

(1) Find the electric field intensity in the coaxial cable. (15%)

(2) Determine the capacitance per unit length of the coaxial cable. (15%)

3. Determine the resistance between two concentric spherical surfaces of radii r_1 and r_2 ($r_1 < r_2$). Here, assume that the space between the surfaces is filled with a homogeneous and isotropic material having a conductivity σ . (20%)

4. Given an air coaxial transmission line that has a solid inner conductor of radius r_i and a very thin outer conductor of inner radius r_o , find the inductance per unit length of the transmission line. (20%)