系所組別：工程科學系甲乙丙丁戊組

考試科目：工程數學　　考試日期：0223，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve $x^2y'' - xy' + y = \cos(\ln x)$. (20%)

2. Solve $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} + e^{-t}\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$ with $\begin{Bmatrix} x \\ y \end{Bmatrix}(0) = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$. (20%)

3. The area enclosed by two vectors $d\vec{r}_1$ and $d\vec{r}_2$ is $dA = \left|d\vec{r}_1 \times d\vec{r}_2\right|$.

   Questions:

   a. The surface $\sum$ is given by $z = z(x, y)$. Derive the area $d\sigma$ of the lateral surface in terms of dx and dy. (5%)

   b. Calculate $\iint_{\sum} (x^2 + y^2)d\sigma$, where the surface is given by $z = 16 - x^2 - y^2$ lying between $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. (15%)

4. Solve $\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = 0$, $1 \le r \le 2$, $0 \le \theta \le 2\pi$, $T(1,\theta) = 0$, $T(2,\theta) = f(\theta)$
   where $f(\theta) = f(-\theta)$. (20%)

5. The complex variable z can be expressed as $z = x + iy$ or $z = re^{i\theta}$, where $i = \sqrt{-1}$.

   Questions :

   (a) Show that $\sin^2 z + \cos^2 z = 1$. (5%)

   (b) Calculate $\int_0^{2\pi} \frac{2\sin\theta}{2 + \sin^2\theta} d\theta$. (15%)