

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (1) Find a recurrence relation for the number of bit strings of length  $n$  that do not have two consecutive 0s. Note that initial conditions should be given. (5%)  
 (2) How many such bit strings are there of length 5? (5%)  
 (3) Solve the recurrence relation. (5%)
2. (1) Find the smallest relation containing the relation  $\{(1,2), (1,4), (3,3), (4,1)\}$  on  $U = \{1, 2, 3, 4\}$ , that is  
 (a) reflexive, symmetric, and transitive. (5%)  
 (b) Reflexive, antisymmetric and transitive. (5%)  
 (2) Which of these nonplanar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a planar graph?  
 (a)  $K_5$ , (b)  $K_6$ , (c)  $K_{3,3}$ , (d)  $K_{3,4}$  (Hint:  $K_n$  denotes complete graph;  $K_{a,b}$  denotes complete bi-partite graph.) (5%)
3. (1) Given a computer network, as shown below, where the nodes are computer centers, and the weight of each edge is the response time (in seconds) between the two end nodes. Find a route with the shortest response time between the node A and node Z.

	A	B	C	D	E	F	Z
A	0	3	5	-	7	-	-
B	3	0	-	4	5	-	-
C	5	-	0	4	5	6	-
D	-	4	4	0	6	-	-
E	7	5	5	6	0	4	3
F	-	-	6	-	4	0	2
Z	-	-	-	-	3	2	0

- (2) Is there a Hamilton cycle in the computer network? Is there an Euler cycle or Euler path? (10%)
4. (1) Let  $G$  be the grammar with alphabet  $\{0, 1\}$ , starting symbol  $S$ , and productions  $P = \{ S \rightarrow 11S, S \rightarrow 0 \}$ . Describe  $L(G)$ , the language of this grammar. (5%)  
 (2) Given the state transition table as follows. ( $S_0$  is the start state.  $S_0$  and  $S_4$  are "accept" states.)  
 (a) Find the nondeterministic finite-state automaton. (10%)  
 (b) Find a deterministic finite-state automaton that recognizes the same state transition table. (10%)

state	Next state	
	Input = 0	Input = 1
$S_0$	$S_0, S_2$	$S_1$
$S_1$	$S_3$	$S_4$
$S_2$	Not defined	$S_4$
$S_3$	$S_3$	Not defined
$S_4$	$S_3$	$S_3$

5. (1) Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$  and the ordering of elements of  $U$  has the elements in increasing order. That is  $a_i = i$ . Assume that we represent  $U$  as a bit string. We represent a subset  $A$  of  $U$  with the bit string of length 8, where the  $i$ th bit in the string is 1 if  $a_i$  belongs to  $A$  and is 0 if  $a_i$  does not belong to  $A$ . The bit strings for the sets  $M = \{1, 2, 3, 4, 5\}$  and  $N = \{1, 3, 5, 7\}$  are 11111000, and 10101010, respectively. Find the union and intersection of  $M$  and  $N$  and represent them using bit strings. (5%)
- (2) Consistent system specifications should not contain conflicting requirements which would be used to derive a contradiction. Determine if the following specifications are consistent:
- “The message is stored in the router’s buffer or it is sent.”
- “The message is not stored in the router’s buffer.”
- “If the message is stored in the router’s buffer, then it is sent.”
- (Hint: you can express the specifications using logic expressions. Let  $p$  denote “the message is stored in the router’s buffer”. Let  $q$  denote “the message is sent.” Try to find an assignment of  $p$  and  $q$  to make the three specifications to be all true.) (5%)
- (3) A program segment is said to be partially correct with respect to the initial assertion  $p$  and the final assertion  $q$  if whenever  $p$  is true for the input values of  $S$  and  $S$  terminates, then  $q$  is true for the output values of  $S$ . This can be denoted as  $p\{S\}q$ . Consider the program segment “ $a := 3; \quad n := a + m;$ ” given the initial assertion  $p: m := 1$ . Find the final assertion  $q$ . (5%)
6. (1) The algorithm for evaluating a polynomial  $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$  at  $x = c$  is as following:
- Procedure polynomial( $c, a_0, a_1, \dots, a_n$ : real numbers)**
- ```

power := 1
y := a0
for i := 1 to n
begin
    power := power * c
    y := y + ai * power
end.
```
- How many multiplications and additions are used to evaluate a polynomial of degree  $n$  at  $x = c$ ? (Hint: Do not count additions used to increment the loop variable.) (5%)
- (2) A committee of three members decides issues for a company. Each member votes either yes or no for each proposal. A proposal is passed if there are at least two yes votes. Design a logic circuit that determines if a proposal is passed. (5%)