

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Explain what are round-off error and truncation error in numerical methods. (10%)

2. The function is given as $f(x)$, use Taylor series expansion to relate $f(x + \Delta x)$ and $f(x)$. (10%).

Derive the relation: $\frac{df(x)}{dx} = \frac{1}{2\Delta x} [3f(x) - 4f(x - \Delta x) + f(x - 2\Delta x)]$ (10%)

3. Matrix A can be decomposed to be the product $A=LU$, where L is a lower triangular matrix and U is an upper triangular matrix, respectively. Given the matrix $A = \begin{pmatrix} 2 & 4 \\ 4 & 9 \end{pmatrix}$, compute $L=?$ and $U=?$ (20%)

4. Use any curve fitting method to determine a polynomial $p(x)$ of degree 2 such that $p(-1) = 13$, $p(0) = 1$, and $p(1) = -1$. (10%). Determine numerically the integral $\int_{-1}^1 p(x) dx = ?$ and $\frac{d^2 p(0)}{dx^2} = ?$ (10%).

5. Given a nonlinear equation $f(x) = 0$, describe any method you know to find the roots of the equation. (10%),

6. The Gaussian quadrature gives the following formula

$$\int_{-1}^1 f(x) dx = C_0 f(x_0) + C_1 f(x_1)$$

Describe how to determine the corresponding coefficients. (20%)