※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Find the solution of the differential equation

$$\frac{d^3y}{dt^3} - 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} - 6y = e^{-t}, \quad y(0) = \frac{dy}{dt}(0) = \frac{d^2y}{dt^2}(0) = 1 \quad (30\%)$$

2. Find the solution of the partial differential equation

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + f(x,t), \quad 0 \le x \le 1, \ t \ge 0$$

with the conditions

$$\frac{\partial T}{\partial x}(0,t) = \frac{\partial T}{\partial x}(1,t) = 0, \quad T(x,0) = 1. \quad (30\%)$$

3. Evaluate the surface integral $\iint\limits_{\Sigma} x^2 d\sigma$, where Σ is the upper surface of $x^2 + y^2 + z^2 = a^2$ intersected by

the cylinder $x^2 + y^2 = b^2 < a^2$, $z \ge 0$. (20%)

4. (a) $\sin z$ can be expressed as $\sin z = U(x,y) + iV(x,y)$, $z = x + iy$, $i = \sqrt{-1}$. What are

the functions $U(x,y)$ and $V(x,y)$? (10%)

(b) $\sinh z$ can be expressed as $\sinh z = R(x,y) + iQ(x,y)$. What are the functions

$R(x,y)$ and $Q(x,y)$? (10%)