※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

題號 1-20 之題為是非題，內容描述若為正確者，答案請寫”T”，若內容描述為不正確者，答案請寫”F”（用其他符號不予計分）。請依題號順序書寫於答案卷上，並請清楚標示題號(每題 3 分，合計 60 分)。題號 21-24 為計算題，請列出過程。每題 10 分，合計 40 分。

1. Let $Q(x,y)$ denote “x+y=0.” The quantification $\exists y \forall x Q(x,y)$ is true.
2. The set of odd positive integers is a countable set.
3. *x* is covered by *y* (*x* C *y*) in a partially ordered set (P, ≤) if x<y and it is possible to have x < z < y.
4. The transitive closure of the symmetric closure of the reflexive closure of a relation R is an equivalence relation.
5. The statement logically equivalent to $(p \vee q) \to r$ for using only the connectives $\neg$ and $\wedge$ is $\neg(\neg[p \wedge \neg q] \wedge \neg r)$.
6. The number of unordered samples, with replacement, of size r chosen from a population of size n is C(n+r, r).
7. The minimum number of people required is 100 to ensure that at least 9 were born in the same month.
8. A graph G has v vertices and e edges. If it is acyclic, then e $\geq$ (v-1).
9. If directed graph G has an Euler cycle then for any vertex v of G, the in-degree equals the out-degree.
10. Assume a graph with n vertices, m edges, and c connected components. Then the number of (*m-n+c-1*) edges must be removed to produce the spanning forest.
11. Two sorted lists with *m* elements and *n* elements can be merged into a sorted list using no more than *(m+n-1)* comparisons.
12. The self-complementary of a simple graph with fives vertices is the cycle C_5.
13. If G is a connected planar simple graph with e edges and v vertices where $v \geq 3$, then $e \leq (3v-6)$.
14. An undirected graph has an even number of vertices of odd degree.
15. The complete graph K_5 is bipartite.
16. The complete bipartite graph $K_{3,3}$ is planar.
17. A partially ordered set can have more than one maximal element and more than one minimal element.
18. The two simple graphs with the following adjacency matrices are isomorphic.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

19. Assume the relation R on a set is represented by

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Then R is both reflexive and symmetric.

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20. The two graphs with adjacency matrices shown are homeomorphic.

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

21. Given the five alphabet symbols A, B, C, D, and E with the frequencies 2, 3, 15, 25, 55.
    (a) Find a Huffman code for the situation. You should show your derivation.
    (b) Decode the message 01001100010110111011110

22. Find the zero-one matrix of the transitive closure of the relation R where

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

23. A computer system considers a string of decimal digits a valid codeword if it contains an even number of 0 digits. For example, 124067890 is valid, whereas 135086503456 is not valid. Let a_i be the number of valid n-digit codewords. Find a recurrence relation for a_n.

24. For the given nondeterministic finite-state automaton;
(a) Find the state table.
(b) Find the language recognized by the nondeterministic finite-state automaton.