

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (a) Find the complex form of the Fourier series of the function

$$f(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \end{cases} \quad (17\%)$$

(b) Find the particular solution of $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = f(t)$. (8%)

2. Both matrices $[M]$ and $[K]$ are positive definite.

(a) Show that all eigenvalues λ of $\lambda[M]\{x\} = [K]\{x\}$ are positive (15%)

(b) Show that if λ_1 and λ_2 are distinct, then $\{x\}_1^T [M] \{x\}_2 = 0$ and $\{x\}_1^T [K] \{x\}_2 = 0$. (10%)

3. (a) The wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$, $-\infty \leq x \leq \infty$, $t \geq 0$ has the initial conditions

$$u(x, 0) = g(x) \quad \text{and} \quad \frac{\partial u}{\partial t} = h(x).$$

Write in details to obtain the general solution. (20%)

(b) Using result of part (a) to find the solution for the initial conditions

$$u(x, 0) = \sin x \quad \text{and} \quad \frac{\partial u}{\partial t} = \cos x. \quad (5\%)$$

4. (a) Verify the function $u(x, y) = x^2 - 2xy - y^2$ is harmonic in the entire complex plane. (5%)

(b) Find v the conjugate harmonic function of u . (10%)

(c) Express $f(z) = u + iv$ in terms of z , where $z = x + iy$. (10%)