※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

每題 10 分，共 10 題，合計 100 分。若題中有小題，該題分數由各小題平分，例如：第 3 題有四小題，則每一小題為 2.5 分。
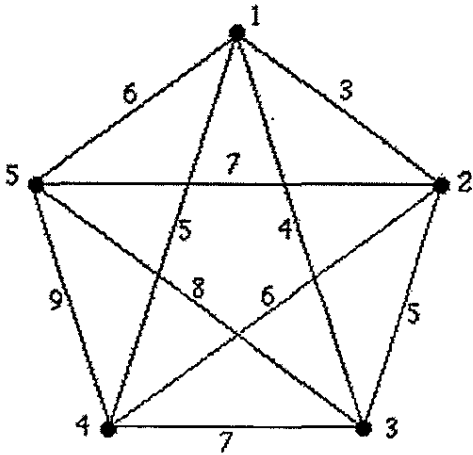
1. (a) Find the number of permutations of 1, 2, . . . , 8 that begin with 52 or end with 387.

   (b) Find the number of permutations of 1, 2, . . . , 8 that begin with 52 or end with 327.

2. Let R be the relation on the set of words in the English language where $x\,R\,y$ if x precedes (that is, comes before) y in the dictionary. Show that R is not a partial ordering.

3. Represent the postfix expression ABC**CDE+/- as

   (a) A binary tree;

   (b) The prefix form;

   (c) The usual infix form; and

   (d) The fully parenthesized infix form.

4. Use a loop invariant to prove that the following program segment for computing nx (x a real number), where n is a positive integer, is correct.

   > $value := 0$
   >
   > $\qquad i := 1$
   >
   > **while** $\ i \leq n$
   >
   > $\qquad$ **begin**
   >
   > $\qquad value := value + x$
   >
   > $\qquad i := i + 1$
   >
   > $\qquad$ **end**

5. Solve the recurrence relation $a_n = a_{n-1}^2 / a_{n-2}$ if $a_0 = 1$ and $a_1 = 2$.

6. (a) What is a Hamiltonian cycle?

   (b) What is an Euler cycle?

   (c) Give an example of graph that has a Hamiltonian cycle but not an Euler cycle.

   (d) Prove that the graph has the specified properties.

7. Prove that if A is the adjacency matrix of a simple graph, the ij^{th} entry of A^n is equal to the number of paths of length n from vertex i to vertex j, n=1, 2,...... .

8. Let $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ be a second-order linear homogeneous recurrence relation with constant coefficients. If a is a sequence defined as above statement, and $a_0 = U_0$, $a_1 = U_1$. Prove that if r_1 and r_2 are the roots of $t^2 - c_1 t - c_2 = 0$ with $r_1 \neq r_2$, then there exist constants b and d such that $a_n = b r_1^n + d r_2^n$, $n = 0,1,2,\dots$.

9. Suppose the vertices of K_5 are numbered 1, 2, 3, 4, 5 (in clockwise order) and each edge is assigned a weight equal to the sum of the labels on the endpoints of the edge, as in the following figure. Find a spanning tree of minimum weight for this graph.

10. (a) What is the Backus-Naur form of the grammar described as follows:

    1. a sentence is made up of a noun phrase followed by a verb phrase or else by a noun phrase followed by a verb phrase followed by a noun phrase.

    2. a noun phrase is made up of a noun, an adjective followed by a noun, or an article followed by a noun.

    3. a verb phrase is made up of a verb.

    4. articles are *a* and *the*.

    5. adjectives are *lengthy*, *boring*, and *inaccurate*.

    6. nouns are *book*, *newspaper*, and *information*.

    7. verbs are *reads* and *contains*.

    (b) Explain how "the book contains lengthy information" can be obtained.