※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve $x^3\dfrac{d^3y}{dx^3}+2x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+y=\dfrac{1}{x}$ 　　　　(25%)

2. Solve $y$ from the differential equation

$$\frac{d}{dt}\begin{Bmatrix}x\\y\end{Bmatrix}+\begin{bmatrix}3&1\\1&3\end{bmatrix}\begin{Bmatrix}x\\y\end{Bmatrix}=e^{-t}\begin{Bmatrix}1\\-1\end{Bmatrix} \quad \text{with} \quad \begin{Bmatrix}x\\y\end{Bmatrix}(0)=\begin{Bmatrix}1\\1\end{Bmatrix} \quad (25\%)$$

3. Solve the partial differential equation $\dfrac{\partial^2 T}{\partial x^2}+\dfrac{\partial^2 T}{\partial y^2}=f(x,y)$, $0\le x,y\le a$,

   with the boundary conditions $T(0,y)=T(a,y)=\dfrac{\partial}{\partial y}T(x,0)=\dfrac{\partial}{\partial y}T(x,a)=0$ (25%)

4. a) Calculate $\displaystyle\sum_{j=0}^{2m-1}\cos(x_j)$, where $x_j=-\pi+(\dfrac{j}{m})\pi$ 　(10%)

   b) Calculate $\displaystyle\int_0^\infty \frac{\cos(mx)}{(1+x^2)(4+x^2)}dx$ 　　(15%)