編號： 120

國立成功大學 108 學年度碩士班招生考試試題

系　　所：工程科學系

考試科目：計算機數學

考試日期：0224，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Let the set of integers denotes by **Z**. Which of the following relations on the set of all functions from **Z** to **Z** are equivalence relations? Determine the properties of an equivalence relation that the others lack. (10%)
   (a) { (f, g) | f(1) = g(1)}
   (b) { (f, g) | f(0) = g(1) and f(1) = g(0)}
   (c) { (f, g) | f(x) - g(x) = 1 for all x∈ **Z**}
   (d) { (f, g | f(x0 - g(x) =C for some C∈ **Z** for all x∈ **Z** }
   (e) { (f, g) | f(0) = g(0) or f(1) = g(1)}

2. (a) What is the second principle of mathematical induction? (5%)
   (b) Use the second principle of mathematical induction to show that if n is an integer greater than 1, then n can be written as the product of primes. (5%)

3. A string that contains only 0s, 1s, and 2s is called a ternary string.
   (a) Find a recurrence relation for the number of ternary strings that do not contain two consecutive 0s. (4%)
   (b) What are the initial conditions? (3%)
   (c) How many ternary strings of length 6 do not contain two consecutive 0s? (3%)

4. (a) Show that if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equal to 9. (5%)
   (b) Is the conclusion in (a) true if four integers are selected rather than five? Explain your answer. (5%)

5. Find the zero-one matrix of the transitive closure of the relation R where the relation is represented by matrix

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}. \quad (5\%)$$

6. Show that the "greater than or equal" relation (≥) is a partial ordering on the set of integers. (5%)

7. Let G = (V, E) be a simple graph. Let R be the relation on V consisting of pairs of vertices (u, v) such that there is a path from u to v or such that u = v. Show that R is an equivalence relation. (10%)

8. Show that there is a simple path between every pair of distinct vertices for a connected undirected graph. (10%)

國立成功大學 108 學年度碩士班招生考試試題

系　　所：工程科學系

考試科目：計算機數學

考試日期：0224，節次：3

9. Assume 7 courses called A to G, will be taken the final exams. Suppose that the following pairs of courses have common students: A and B, A and C, A and D, A and G, B and C, B and D, B and E, B and G, C and D, C and F, C and G, D and E, D and F, E and F, E and G, F and G. How can the final exams be scheduled so that no student has two exams at the same time? (15%)

10. Find a determine finite-state automaton that recognizes the same language as the following nondeterministic finite-state automaton. (15%)