

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Let $x(t)$ be a signal with Fourier transform $X(j\omega)$. Please express the inverse Fourier transform of

$$Y(j\omega) = \frac{dX(j\omega)}{d\omega}$$

in terms of $x(t)$.

2. (15%) We know that the Fourier transform of $x(t) = e^{-at}u(t)$ is $X(j\omega) = 1/(a + j\omega)$ where $a > 0$ and $u(t)$ is the unit step function. Please find the inverse Fourier transform of

$$Y(j\omega) = \frac{1}{(a + j\omega)^n}$$

where n is a positive integer.

3. (15%) Let

$$x(t) = \begin{cases} 1, & |t| \leq 1 \\ 0, & |t| > 1. \end{cases}$$

- a. (5%) Please find the spectrum $X(j\omega)$ of $x(t)$.

- b. (10%) Find the value of

$$\int_0^{\infty} \frac{\sin(\omega)}{\omega} d\omega.$$

4. (15%) We know that the Fourier transform of $u(t)$ is $1/(j\omega) + \pi\delta(\omega)$ where $\delta(\omega)$ is the impulse function. For a signal $x(t)$ with Fourier transform $X(j\omega)$, express the Fourier transform of

$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

in terms of $\delta(\omega)$ and $X(j\omega)$.

5. (15%) Consider a continuous-time system with input $x(t)$ and output $y(t)$ related by $y(t) =$

$$\sqrt{|x(\tan^{-1}(t))|} \text{ where } -\pi/2 \leq \tan^{-1}(t) \leq \pi/2.$$

- a. (3%) Is this system stable?

- b. (3%) Is this system causal?

- c. (3%) Is this system invertible?

- d. (3%) Is this system linear?

- e. (3%) Is this system time-invariant?

6. (10%) Let $z[n] = x[n] * y[n]$ where '*' denotes the convolution and both of $x[n]$ and $y[n]$ are odd signals. Is $z[n]$ an odd signal or an even signal? The proof is required.

7. (10%) Let $z[n] = x[n] * y[n]$. Is $z[n - 7] = x[n - 3] * y[n - 4]$? The explanation is required.

8. (10%) Can this frequency-domain signal

$$\frac{\sin(10\omega/2)}{\sin(\omega/2)}$$

be a valid discrete-time Fourier transform of a discrete-time signal? Please provide detailed reasons.