編號： 111

國立成功大學 108 學年度碩士班招生考試試題

系　　所：工程科學系

考試科目：線性代數與機率

考試日期：0224，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) If we have

$$\begin{pmatrix} E & F \\ -F & E \end{pmatrix} = \frac{1}{2}\begin{pmatrix} A & B \\ -B & A \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} C & D \\ -D & C \end{pmatrix}$$

where $A, B, C, D, E,$ and F are all real numbers, please express $E^2 + F^2$ in terms of $A, B, C,$ and D.

2. (10%) For an $m \times n$ matrix $\boldsymbol{A}$, we have $\boldsymbol{AA}^T$ is an identify matrix. Is $m \leq n$ or $n < m$? Please provide detailed reasons.

3. (20%) Let $Y = X^2$ where X is a normal random variable with parameters 0 and σ^2, that is, $X \sim N(0, \sigma^2)$.
   a. (10%) Please find the probability density function of Y.
   b. (5%) Find the moment-generating function of X.
   c. (5%) Find the variance $\mathrm{Var}(Y)$ of Y.

4. (20%) Let X be an exponential random variable with mean $1/\lambda$. Also let $Y = [X] + 1$ where $[X]$ denotes the integer part of X, e.g., $[3.4] = 3$ and $[2] = 2$.
   a. (10%) Find the moment-generating function of Y.
   b. (5%) Find the mean $\mathrm{E}(Y)$ of Y.
   c. (5%) Find the variance $\mathrm{Var}(Y)$ of Y.

5. (10%) Let X be a negative binomial random variable with parameters (r, p). Find the moment-generating function of X.

6. (15%) Let X be a Poisson random variable with parameter λ>0.
   a. (10%) Find $\mathrm{E}(X^3)$.
   b. (5%) Find $\mathrm{E}(X!)$ if $\lambda < 1$.

7. (10%) Let X and Y be jointly normal random variables with $\mathrm{Var}(X) = \mathrm{Var}(Y)$. Let random variables $W = X + Y$ and $Z = X - Y$. Are W and Z independent? Please provide detailed reasons.