

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Find the nullity of the binary matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

2. (10%) Boxes A, B, and C contain 3 yellow balls and 5 red balls, 2 yellow balls and 6 red balls, 3 yellow balls and 1 red ball, respectively. One ball is selected at random from each box. Then, 2 of the 3 balls are red. Find the probability that the ball selected from box B is red.

3. (10%) For a geometric random variable X , show that

$$P(X = k + n | X > n) = P(X = k)$$

where k and n are positive integers.

4. (10%) For two random variables X and Y , express the covariance $\text{Cov}(X - Y, X + Y)$ in terms of $\text{Var}(X)$ and $\text{Var}(Y)$.

5. (40%) Let the joint probability density function of X and Y be given by

$$f(x, y) = \begin{cases} cxy, & \text{if } 0 \leq y \leq x \leq 1; \\ 0, & \text{otherwise} \end{cases}$$

where c is a real number.

- (5%) Find c .
 - (10%) Calculate the marginal probability density functions of X and Y , respectively.
 - (10%) Find the mean $E(Y)$ of Y and the mean $E(X)$ of X .
 - (5%) Find $E(XY)$.
 - (5%) Are X and Y independent?
 - (5%) Are X and Y uncorrelated?
6. (10%) Let X_1, X_2 , and X_3 be independent exponential random variables with means 1, 1/2, and 1/3, respectively. Also let $X = \min(X_1, X_2, X_3)$. Find the cumulative distribution function of X .
7. (10%) Let X be a Gaussian random variable with parameters $(0, \sigma^2)$. Let $Y = X^2$.
- (5%) Find the moment-generating function of X .
 - (5%) Find the variance of Y .