

國立成功大學
110學年度碩士班招生考試試題

編 號： 119

系 所： 工程科學系

科 目： 計算機數學

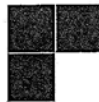
日 期： 0203

節 次： 第 3 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. You roll 2 dice. What is the probability that the two values that turn up are different?
You roll 3 dice. What is the probability that the three values that turn up are all different?
What is the probability that there are at least two dice with the same value?
You roll n dice. What is the probability that there are at least two dice with the same value?
(15%)
2. Given a prime number p . Show that $\binom{p}{k}$ is divisible by p for all $0 < k < p$.
Does this also hold when p is not prime? Justify. (10%)
3. How many triples (x, y, z) with $x, y, z \in \mathbb{N}^+$ are there such that $x + y + z = 10$? How many are there such that $x + y + z = N$, for an arbitrary N ? (10%)
4. Suppose that you live in a country that only has two kinds of stamps: stamps that cost \$4 and stamps that cost \$5. Sending a letter may cost any amount from \$12 upwards. Show that you can always use \$4 and/or \$5 stamps to get to any amount ≥ 12 you have to pay for a letter. (15%)
5. Take a look at the following shape of a puzzle piece:



Now, consider a grid of 1024×1024 squares, of which we remove 1 square. Show that the rest of the grid can be completely covered by pieces of the shape above (without any piece sticking out on the side, or covering the square we removed).

(Hint: Prove that you can do this for any grid of dimensions $2^n \times 2^n$, by induction over n .)
(15%)

6. Let X be a random variable with PDF given by

$$f_X(x) = \begin{cases} cx^2 & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the constant c . (5%)
(b) Find EX and $\text{Var}(X)$. (10%)
(c) Find $P\left(X \geq \frac{1}{2}\right)$. (5%)

7. Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} x^2 \left(2x + \frac{3}{2}\right) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

If $Y = \frac{3}{X} + 1$, find $\text{Var}(Y)$. (15%)