

國立成功大學  
110學年度碩士班招生考試試題

編 號：110

系 所：工程科學系

科 目：線性代數與機率

日 期：0203

節 次：第 3 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) Express the determinant of

$$A = \begin{pmatrix} x_1 & x_2 & x_3 \\ x_1^2 & x_2^2 & x_3^2 \\ x_1^3 & x_2^3 & x_3^3 \end{pmatrix}$$

in terms of  $x_1, x_2$ , and  $x_3$ .

2. (15%) For two random variables  $X$  and  $Y$  with variances  $\text{Var}(X) = 5$  and  $\text{Var}(Y) = 3$ , respectively. If the covariance  $\text{Cov}(X, Y) = 0$ , then find the correlation coefficient  $\rho(X + Y, X - Y)$ .

3. (15%) If  $X$  is a Poisson random variable with  $P(X = 1) = P(X = 3)$ . Find  $\max_n P(X = n)$  and

$$\hat{n} = \operatorname{argmax}_n P(X = n).$$

4. (10%) Let  $N$  be an integer-valued random variable with mean  $E(N) = 5$  and  $X_i$ 's be independent random variables with equal mean  $E(X_i) = 10$ . If  $N$  is independent of  $X_i$ 's, then find

$$E\left(\sum_{i=1}^N X_i\right).$$

5. (15%) Let  $X$  and  $Y$  be independent and identically distributed Gaussian random variables. Show that random variables  $U = X + Y$  and  $V = X - Y$  are independent.

6. (10%) Let  $X$  and  $Y$  be independent random variables with  $E(X) = 1$ ,  $E(Y) = 3$ ,  $\text{Var}(X) = 1$ , and  $\text{Var}(Y) = 4$ . Find  $\text{Var}(XY)$ .

7. (20%) Let  $X$  be a Poisson random variable with  $P(X = 0) = e^{-2}$ .

a. (10%) Find  $P(X > 2)$ .

b. (10%) Find  $E(X^3)$ .